CSIR MATHS QUESTION PAPER
28- July- 2025

Section :- PART - A:-

Qus 1: Three friends, Mr. Rahman, Mr. George and Mr. Vedant, met after a long time. They were wearing red, green and violet colour shirts. Mr. Rahman and the person wearing violet shirt noticed that none of the three is wearing a colour that starts with same letter as him name. Which one of the following is the correct match of the persons with the colour of their shirts?

(1) Rahman-Violet, George-Red, Vedant-Green
(2) Rahman-Green, George-Violet, Vedant-Red
(3) Rahman-Green, George-Red, Vedant-Violet
(4) Rahman-Red, George-Violet, Vedant-Green

Qus 2: Kavita starts from her house and walks 200m northward, then turns 45° right and walks 70m. After that, she turns 90° right and wallks 70m. Which of the following is the closest value of the shortest distance between Kavita's current location and her house?

(1) 296m
(2) 240m
(3) 200m
(4) 223m

Qus 3: A stock market trader has two thirds of her investment on a day. Next day she recovered one third of the previous day's loss. What fraction of her initial investment is she left with?

(1) 1/3 (2) 2/3
(3) 2/9 (4) 5/9

Qus 4: The value of a company is measured as the total value of its shares owned by different investors. Rakesh owns 2/15 of the shares of a company. He sells 1/3 of his shares of Rs. 75,000/-. What is the total value of the company at that time?

(1) Rs. 15,75,800
(2) Rs. 16,87,500
(3) Rs. 17,75,800
(4) Rs. 18,27,500

Qus 5: A number is mistakenly divided by 2 instead of being multiplied by 2. What is the change in the result caused by this mistake?

(1) 25%
(2) 50%
(3) 75%
(4) 100%

Qus 6: Number of Rose, Lotus, and Marigold plants in a garden are in the proportion 8:5:7. Later, 75%, 40%, and 50% more plants of their respective categories were added. What will be the new proportion of plants, in the same order?

(1) 5:3:4
(2) 4:2:3
(3) 5:4:3
(4) 7:4:5

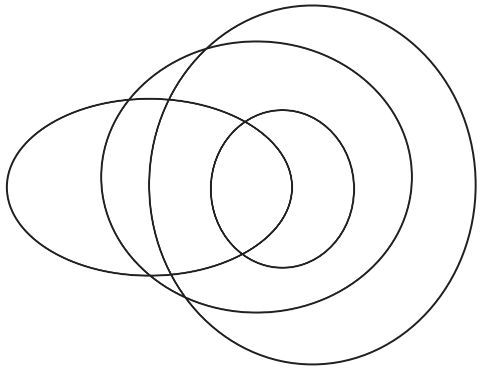
Qus7: Suresh asked Ramesh to identify the person in a photo that the latter is holding. Ramesh responds. "I have no brother or sisters. However, that man's father is my father's son." Who is the person in the photo?

(1) Suresh
(2) Ramesh
(3) Ramesh's son
(4) Ramesh's Cousin

Qus 8: Sum of the digits of a two-digits number 'ab' is subtracted from the number and the result is divided by 9. Then the result of this will be

(1) Always a
(2) Always b
(3) Neither a nor b
(4) Either a or b depending on a + b

Qus 9: The following diagram represents the relationship between four categories.



The categories could be

(1) Rivers, water bodies, oceans, sources of evaporation
(2) Parliamentarians, celebrities, elected persons, professional politicians
(3) Monkeys, four-legged animals, pet animals, land animals
(4) Fumiture, chairs, seats, wooden objects

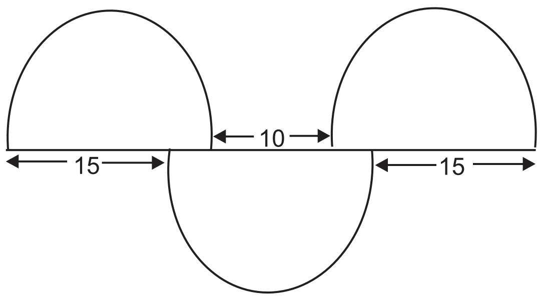
Qus 10: In a code, the word DELTOID is written as 3152893. Then LOTION could be written as

(1) 582986 (2) 582981
(3) 198396 (4) 198392

Qus 11: A cylindrical container of radius 20 cm was filled with water up to 25 cm height. A solid spherical ball of radius 7 cm was then immersed in the water. What would be the approximate increase in water level in the container after the ball was fully immersed?

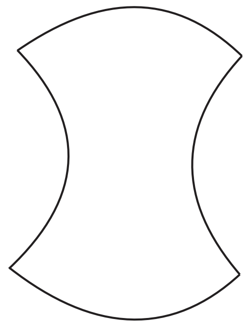
(1) 1.14 cm (2) 2.28 cm
(3) 5.50 cm (4) 7.00 cm

Qus 12: Three identical semi-circles are arranged as shown. What is the diameter of the semi-circles?



(1) 5π (2) 20
(3) $15\pi/2$ (4) 25

Qus 13: A circle of radius 1 unit is divided into four quarters and rejoined as shown below.



What is the area of this shape?

(1) π (2) 1
(3) 2 (4) 4

Qus 14: A car has wheels of diameter 36 cm. If it runs at a speed of 60 km/h, then the rotation per minute (RPM) will be closets to ______.

(1) 884 (2) 898
(3) 906 (4) 986

Qus 15: What will be the digit at the unit's place of

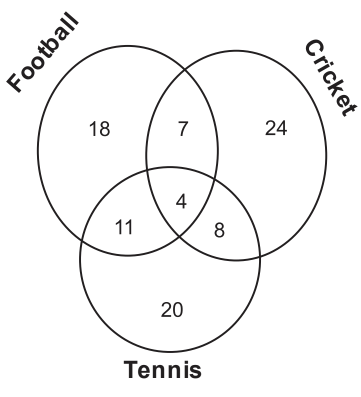
$1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3$?

(1) 0 (2) 5
(3) 7 (4) 9

Qus 16: Consider the following statements:
Statement I: All Booklets are Manuals.
Statement II: All Manuals are Catalogues.
If Statement I and II are True, which one of the following conclusions can be conclusively drawn?

(1) All Manuals are Booklets
(2) All Catalogues are Booklets
(3) All Booklets are Catalogues
(4) All Catalogues are Manuals

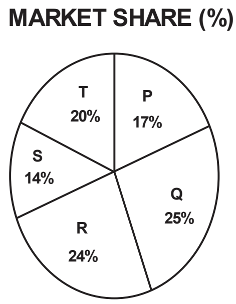
Qus 17: The given Venn diagram shows number of players playing one or more than one sports.

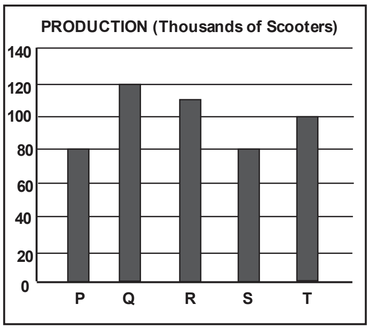


The percentage of players who play exactly two sports is closest to __________%

(1) 5 (2) 14
(3) 28 (4) 32

Qus 18: The market share (%) and annual production of scooters from five automobile companies P,Q,R,S, and T are shown in graphs.





If the profit of a company is directly proportional to the ratio of market share to production, then which of the following statements is/are CORRECT?

Statement X: Companies T and P have same profit

Statement Y: Company R has the maximum profit

Statement Z: Company S has the minimum profit

(1) X and Y (2) X and Z
(3) Y and Z (4) Only Z

Qus 19: The initial monthly salaries of employees John, Riya, and Sunil were in the proportion 4:3:5. After an increase of Rs 10000 monthly to all, the new proportion becomes 6:5:7. What was the initial salary of Sunil?

(1) Rs 20000 (2) Rs 25000
(3) Rs 30000 (4) Rs 35000

Qus 20: Rahul and his father started jogging on a circular track of radius $'r'(r>2)$. Rahul completed one round and stopped. His father got tired half way into the first round and returned to his starting point along a straight line. What is the ratio of the distances convered by Rahul and his father?

(1) $\pi r/(\pi+2)$ (2) $2\pi/(\pi+2)$
(3) 1 (4) 2

Section :-PART - B:-

Qus 21: For each $n\geq 1$, let $f_n:[0,1]\to\mathbb{R}$ be defined as

$$f_n(x)=\begin{cases} nx & \text{if } x\in\left[0,\frac{1}{n}\right] \\ 2-nx & \text{if } x\in\left(\frac{1}{n},\frac{2}{n}\right] \\ 0 & \text{if } x\in\left(\frac{2}{n},1\right] \end{cases}$$

Which of the following statements is true?

(1) $(f_n)_{n\geq 1}$ converges uniformly on $[0,1]$ to a continuous function f.

(2) $(f_n)_{n\geq 1}$ converges pointwise on $[0,1]$ to a discontinuous function f.

(3) $(f_n)_{n\geq 1}$ converges pointwise on $[0,1]$ to a continuous function f.

(4) $(f_n)_{n\geq 1}$ does not converge pointwise on $[0,1]$

Qus 22: Let f be an entire function such that $f(\mathbb{C})\subset\{x+iy\mid y=x+1\}$. Which of the following statements is true?

(1) $|f(z)|\to\infty$ as $|z|\to\infty$

(2) $\frac{f(z)}{z}\to 0$ as $|z|\to\infty$

(3) $z\,f(z)\to 0$ as $|z|\to\infty$

(4) $f(z)\to 0$ as $|z|\to\infty$

Qus 23: Let $\mathbb{F}_5$ denote the field with 5 elements. How many 2×2 matrices with entries in $\mathbb{F}_5$ have rank one?

(1) 125 (2) 144
(3) 145 (4) 480

Qus 24: Let $f:\mathbb{C}\to\mathbb{C}$ be a polynomial map. For $R>0$, let $\gamma_R:[0,1]\to\mathbb{C}$ be the map $t\mapsto \mathrm{Re}^{2\pi it}$. Suppose that there exists

$c \in \mathbb{R}$ such that $\int_0^1 \left|(f \circ \gamma_R)(t)\gamma_R'(t)\right| dt \to c$ as $R \to \infty$.

Which of the following statements is False?

(1) The function $z f(1/z) \to 0$ as $|z| \to \infty$

(2) The function f is constant

(3) $c = 0$

(4) $c > 0$

Qus 25: Let $(\lambda_n)_{n\in\mathbb{N}}$ be the sequence of eigenvalues of the Sturm-Liouville problem

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{\lambda}{x} y = 0,\ 1 < x < e^{2\pi},$$

$$y(1) = 0,\ y\left(e^{2\pi}\right) = 0.$$

Then $\sum_{n=1}^{\infty} \frac{1}{\lambda_n}$ is equal to

(1) $\frac{\pi^2}{12}$ (2) $\frac{2\pi^2}{3}$

(3) $\frac{\pi^2}{4}$ (4) $\frac{\pi^2}{16}$

Qus 26: If $y(x)$ is the solution of the integral equation $y(x) = x^2 + 2\int_0^1 x\,t\,y(t)\,dt$, then which of the following statements is true?

(1) $y(0) + y(1) = \frac{1}{2}$

(2) $y(-1) + y(1) = 1$

(3) $y'(0) + y'(1) = \frac{3}{2}$

(4) $y'(-1) + y'(1) = 3$

Qus 27: Which of the following statements is true?

(1) $p \nmid 1 + (p-1)!$ for some odd prime p

(2) $p \mid (1234)^{p-1} - 1$ for all primes $p > 700$

(3) There exist $a \in \mathbb{Z}$ and a prime $p > 11$ such that $p \nmid a^p - a$

(4) $p \nmid \frac{(p^2)!}{(p!)^2}$ for some odd prime p

Qus 28: Let $S = \{1,2,3,4,5\}$ be equipped with the topology $\tau = \{\emptyset, \{1\}, S\}$. What is the number of homeomorphisms of S onto itself?

(1) 25 (2) 120

(3) 24 (4) 6

Qus 29: Which of the following statements is true?

(1) The ideal $2\mathbb{Z}[i]$ is maximal in $\mathbb{Z}[i]$

(2) The ideal $X\mathbb{C}[X,Y]$ is maximal in $\mathbb{C}[X,Y]$

(3) The set of all polynomials in $\mathbb{C}[X]$ whose coefficients add up to 0 is a maximal ideal in $\mathbb{C}[X]$

(4) The ideal $(\sqrt{2}-1)\mathbb{Z}[\sqrt{2}]$ is maximal in $\mathbb{Z}[\sqrt{2}]$.

Qus 30: Let $f(x) = x\log_e\left(1 + \frac{1}{x}\right)$ for $x \in (0, \infty)$.

Which of the following statements is true?

(1) f is unbounded

(2) f is increasing

(3) $\lim_{x\to\infty} f(x) = 2$

(4) f is decreasing

Qus 31: A mobile manufacturing company uses two brands of batteries for its mobiles. The life (in years) of batteries of Brand I follows an exponential distribution with the probability density function

$$f(x) = \begin{cases} e^{-x}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

and that of Brand II follows a gamma distribution with the probability density function

$$g(x) = \begin{cases} \frac{x}{4}e^{-x/2}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

The company uses the batteries of Brands I and II in proportion of 20% and 80% respectively, in its mobiles. The probability that a randomly selected mobile has the battery life more that 2 years is

(1) $\frac{13}{5}e^{-2}$

(2) $\frac{1}{5}\left(e^{-2}+2e^{-1}\right)$

(3) $\frac{1}{5}\left(e^{-2}+8e^{-1}\right)$

(4) $\frac{1}{5}\left(4e^{-2}+2e^{-1}\right)$

Qus 32: Suppose $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim N_3\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right)$ and

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & 0 & 4 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}.$$

Then the partial correlation coefficient $\rho YZ.X$ is

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{3}{4}$ (4) 0

Qus 33: Let A be a subring of the field of rationals $\mathbb{Q}$ such that for any nonzero rational $r \in \mathbb{Q}$, $r \in A$ or $1/r \in A$. Which of the following statements is FALSE?

(1) The set $\left\{a \in A : \frac{1}{a} \notin A\right\} \cup \{0\}$ is an additive subgroup of $\mathbb{Q}$

(2) A has at most one maximal ideal

(3) If $A \neq \mathbb{Q}$, then A has infinitely many prime ideals

(4) For any nonzero $a, b \in A$, a divides b or b divides a in A

Qus 34: Which of the following polynomials is the characteristic polynomial of a real 2×2 matrix A such that $\text{trace}(A) = 7$ and $\text{trace}(A^2) = 29$?

(1) $t^2 + 7t + 10$

(2) $t^2 - 7t + 29$

(3) $t^2 - 7t - 10$

(4) $t^2 - 7t + 10$

Qus 35: Let $u = u(x, y)$ be the solution to the Cauchy problem

$$(y+u)\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x - y, \ x \in \mathbb{R}, y > 0$$

$$u(x,1) = 1 + x \ , \quad x \in \mathbb{R}$$

Then which of the following statements is true?

(1) $u(1,1) = 2$

(2) $u(2,2) = 4$

(3) $u(3,3) = \frac{3}{2}$

(4) $u(4,4) = \frac{2}{3}$

Qus 36: Suppose the distribution of X given θ is normal with mean θ and variance 15. Further, let the prior (improper) distribution of θ be proportional to $1, -\infty < \theta < \infty$. If the observed value of X is 13, then which of the following statements is true?

(1) Posterior mean = Maximum likelihood estimate of θ, Posterior variance $= Var(X \mid \theta)$

(2) Posterior mean = Maximum likelihood estimate of θ, Posterior variance $< Var(X \mid \theta)$

(3) Posterior mean > Maximum likelihood estimate of θ, Posterior variance $= Var(X \mid \theta)$

(4) Posterior mean > Maximum likelihood estimate of θ, Posterior variance $< Var(X \mid \theta)$

Qus 37: Let X be a random sample of size 1 from the probability density function

$$f(x\mid\theta)=\begin{cases}\frac{3}{\theta^3}(\theta-x)^2, & \text{if } 0<x<\theta\\ 0, & \text{otherwise}\end{cases}$$

If $\left(\frac{X}{1-\lambda_1},\frac{X}{1-\lambda_2}\right)$ is a confidence interval for θ with confidence coefficient $1-\alpha$ where $\lambda_i\in(0,1)$, $i=1,2,\lambda_1<\lambda_2$ and $\alpha\in(0,1)$, then which of the following statements is true?

(1) $\lambda_2^2-\lambda_1^2=1-\alpha$

(2) $\lambda_2^3-\lambda_1^3=1-\alpha$

(3) $\lambda_2^2-\lambda_1^2=4(1-\alpha)$

(4) $\lambda_2^3-\lambda_1^3=9(1-\alpha)$

Qus 38: Let p,q be non-negative integers. Consider the following statements:

(A) There is an integer $k\geq 1$ such that
$$p+k=q$$

(B) There is an integer $k\geq 1$ such that
$$q+k=p$$

Which of the following statements is true?

(1) There exist non-negative integers p,q such that both (A) and (B) are true

(2) Both (A) and (B) are false if and only if $p=q$

(3) For all non-negative integers p and q, (A) or (B) is true

(4) There exists $p\neq q$ such that both (A) and (B) are false

Qus 39: Suppose a dynamical system has the Lagrangian

$$L=\left(\dot{q}_1\right)^2+\left(\dot{q}_2\right)^2+\left(q_1\right)^2+\dot{q}_1\dot{q}_2$$

If p_1 and p_2 are momenta conjugate to q_1 and q_2, respectively, then which of the following statements is true?

(1) $\dot{p}_1=2q_1,\dot{p}_2=0$

(2) $\dot{p}_1=-q_1,\dot{p}_2=0$

(3) $\dot{p}_1=-\frac{q_1}{2},p_2=q_2$

(4) $\dot{p}_1=q_1,p_2=-q_2$

Qus 40: Let $y(x)$ be the extremal of the functional

$J[y]=\int_0^{\frac{\pi}{4}}\left((y')^2-4y^2+2xy\right)dx$ subject to $y(0)=0, y\left(\frac{\pi}{4}\right)=1$. Then $y(x)$ is equal to

(1) $\left(1-\frac{\pi}{4}\right)\sin(2x)+x$

(2) $\left(1-\frac{\pi}{16}\right)\sin(2x)+\frac{x}{4}$

(3) $\left(1+\frac{\pi}{4}\right)\sin(2x)-x$

(4) $\left(1+\frac{\pi}{16}\right)\sin(2x)-\frac{x}{4}$

Qus 41: Let $Z_1,Z_2,\ldots$ be a sequence of independent and identically distributed random variables having discrete uniform distribution over $\{1,2,\ldots,2024\}$. Let $Y_n=\sum_{i=1}^{n}Z_i,\ n\geq 2$. Further, let X_n be the remainder when Y_n is divided by 2025. Then, which of the following statements is true?

(1) $\lim_{n\to\infty}P(X_n=0)=\frac{1}{2026}$

(2) $\lim_{n\to\infty}P(X_n=0)=\frac{1}{2025}$

(3) $\lim_{n\to\infty}P(X_n=0)=\frac{1}{2024}$

(4) $\lim_{n\to\infty}P(X_n=0)=\frac{1}{2023}$

Qus 42: Let A,B be non-empty subsets of $\mathbb{N}$ with cardinality $|A|\geq 2$. Let

$S_1=\{f:A\to B\mid f \text{ is one-to-one}\}$ and

$S_2 = \{ g : B \to A \mid g \text{ is onto} \}$

Which of the following statements is true?

(1) If $A \underset{\neq}{\subset} B$ and B is finite, then there is a one-to-one map from S_2 to S_1

(2) If $B = \mathbb{N}$, then there exists a one-to-one map from S_2 to B

(3) If $B = \mathbb{N}$ and A is finite, then there exists a one-to-one map from B to S_1

(4) If A is finite, then S_2 is finite for any B

Qus 43: If the function $s : [0,4] \to \mathbb{R}$ defined by

$$s(x) = \begin{cases} a(x-2)^2 + b(x-1)^2, & 0 \le x \le 1, \\ (x-2)^2, & 1 < x \le 3, \\ 2c(x-2)^2 + (x-3)^3, & 3 < x \le 4 \end{cases}$$

is a cubic spline, then the value of $2a + b + 2c$ is

(1) 2 (2) 3

(3) 4 (4) 5

Qus 44: Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the common probability density function

$$f(x \mid \theta) = \begin{cases} \dfrac{2^\theta \theta}{x^{\theta+1}}, & \text{if } x > 2 \\ 0, & \text{otherwise} \end{cases}$$

where $\theta (> 0)$ is an unknown parameter. Suppose $P\left(Y > \chi^2_{m,\beta}\right) = \beta$, where $Y \sim \chi^2_m$. For testing $H_0 : \theta = 1$ against $H_1 : \theta > 1$, a uniformly most powerful test of size $\alpha, 0 < \alpha < 1$, will reject H_0 if

(1) $\sum_{i=1}^{n} \ln X_i > \frac{1}{2} \chi^2_{2n,\alpha} + n \ln 2$

(2) $\sum_{i=1}^{n} \ln X_i < \frac{1}{2} \chi^2_{2n,1-\alpha} + n \ln 2$

(3) $\sum_{i=1}^{n} \ln X_i > \frac{1}{2} \chi^2_{n,\alpha} + n \ln 2$

(4) $\sum_{i=1}^{n} \ln X_i < \frac{1}{2} \chi^2_{n,1-\alpha} + n \ln 2$

Qus 45: Let $V = \{ ax^3 + bx^2 + cx \mid a, b, c \in \mathbb{R} \}$. For $f \in V$, define $Q(f) = \int_{-1}^{1} \left(f'(t) \right)^2 dt$, where f' denotes the derivative of f. Which of the following statements is FALSE?

(1) Q is a positive definite quadratic form on V

(2) Q takes every positive real value

(3) $Q(x) = 2$

(4) For all $f, g \in V, Q(f+g) = Q(f) + Q(g)$

Qus 46: Let $u = u(x,t)$ be the solution of

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \quad x \in \mathbb{R}, t > 0,$$
$$u(x,0) = 1 + x^2, \ x \in \mathbb{R},$$
$$\frac{\partial u}{\partial t}(x,0) = x + 1, \ x \in \mathbb{R}.$$

Then the value of $u(1,1)$ is

(1) 2 (2) 3

(3) 4 (4) 5

Qus 47: Consider the real matrix

$$A = \begin{pmatrix} 29 & 0 & 55 & 17 \\ 1 & 28 & 46 & 26 \\ 17 & 13 & 33 & 38 \\ 21 & 67 & 0 & 13 \end{pmatrix}.$$

What is the largest real eigenvalue of A?

(1) 101 (2) 67

(3) 103 (4) 113

Qus 48: If $\varphi(x) = x$ is a solution of the ordinary differential equation (ODE)

$$\frac{d^2 y}{dx^2} - \left(\frac{2}{x^2} + \frac{1}{x} \right) \left(x \frac{dy}{dx} - y \right) = 0, \ 0 < x < \infty$$

then the general solution of the ODE is given by

(1) $\left(a + be^{-2x} \right) x, a, b \in \mathbb{R}$

(2) $\left(a+be^{2x}\right)x, a, b \in \mathbb{R}$

(3) $ae^{x}+bx, a, b \in \mathbb{R}$

(4) $\left(a+be^{x}\right)x, a, b \in \mathbb{R}$

Qus 49: Let X be the $\mathbb{R}$-vector space of all twice differentiable real valued functions on $[0,1]$. Consider the linear map $\phi : X \to \mathbb{R}^3$ defined by $\phi(f) = \left(f(1), f'(1), f''(1)\right)$. Which of the following statements is true?

(1) The dimension of $X/\ker\phi$ is 3

(2) $\ker\phi$ is finite dimensional

(3) The dimension of $X/\ker\phi$ is 1

(4) X is finite dimensional

Qus 50: Let X be the image of the interval $[0,1]$ under the Mobius transformation $f(z) = \frac{z-i}{z+i}$. Which of the following statements is true?

(1) X is the line segement joining -1 and $-i$

(2) $X = \left\{ e^{i\theta} \mid \theta \in \left[\pi, \frac{3\pi}{2}\right] \right\}$

(3) X is the line segement joining -1 to 1

(4) $X = \left\{ e^{i\theta} \mid \theta \in \left[-\frac{\pi}{2}, \pi\right] \right\}$

Qus 51: Consider the multiple linear regression model $y_i = \beta_0 + \beta_1 x_{1i} + ... + \beta_8 x_{8i} + \in_i$, $i = 1, 2, ..., 29$ where $\in_1, \in_2, ..., \in_{29}$ are independent and identically normal distributed with mean 0 and variance σ^2. Suppose the model is fitted using the method of least squares. If the calculated value of the F-statistic for testing the significance of regression is 2.50, then the posssible values of R^2 and Adjusted R^2 are respectively.

(1) 0.30 and 0.10
(2) 0.50 and 0.30
(3) 0.50 and 0.16
(4) 0.30 and -0.10

Qus 52: Let

$$A = \left\{ \frac{p}{q} \in (0,1) : p \in \mathbb{N}, q = 2^n \text{ for some } n \in \mathbb{N} \cup \{0\}, \gcd(p,q) = 1 \right\},$$

$$B = \left\{ \frac{p}{q} \in (0,1) : p \in \mathbb{N}, q = 2^n 5^m \text{ for some } n, m \in \mathbb{N} \cup \{0\}, \gcd(p,q) = 1 \right\},$$

$$C = \left\{ \frac{p}{q} \in (0,1) : \frac{p}{q} \text{ has terminating decimal expansion} \right\}$$

be subsets of $(0,1)$. Which of the following statements is true?

(1) $A \underset{\neq}{\subset} C$ and $B \underset{\neq}{\subset} C$

(2) $A \underset{\neq}{\subset} C \underset{\neq}{\subset} B$

(3) $A \underset{\neq}{\subset} B \underset{\neq}{\subset} C$

(4) $A \underset{\neq}{\subset} B = C$

Qus 53: Suppose we want to estimate the population mean $\overline{Y}$ of a variable for a finite population of size 85, with 34 Statisticians and 51 Biologists. We consider the following sampling scheme:
A stratified random sample with 2 strata of Statisticians (Stratum-1) and Biologists (Stratum-2), where 12 Statistician and 15 Biologists are drawn from Stratum-1 and Stratum-2, respectively, using SRSWOR scheme. Denote $\overline{y}S, \overline{y}B$ and $\overline{y}$ as the mean of the variable among the Statistician sample, Biologist sample, and the combined sample, respectively. Which of the following is an unbiased estimator of $\overline{Y}$?

(1) $\overline{y}$

(2) $\frac{2\overline{y}S + 3\overline{y}B}{5}$

(3) $\frac{4\overline{y}S + 5\overline{y}B}{9}$

(4) $\frac{\frac{\overline{y}S}{12} + \frac{\overline{y}B}{15}}{\frac{1}{12} + \frac{1}{15}}$

Qus 54: Let $f:\mathbb{R}\setminus\mathbb{Q}\to\mathbb{R}\setminus\mathbb{Q}$ be the function defined as $f(x)=\dfrac{3x+2}{4x+3}$.

Let $x_1\in\mathbb{R}\setminus\mathbb{Q}$. For $n\geq 1$, define $x_{n+1}=f(x_n)$.

Suppose that the sequence $(x_n)_{n\geq 1}$ converges to a real number ℓ. Which of the following statements is true?

(1) If ℓ is positive, then $\ell=\dfrac{\sqrt{3}}{2}$

(2) If ℓ is positive, then $\ell=\dfrac{1}{\sqrt{2}}$

(3) If ℓ is negative, then $\ell=-\dfrac{\sqrt{3}}{2}$

(4) If ℓ is negative, then $\ell=-\dfrac{1}{2}$

Qus 55: Solve the following linear programming problem

maximize $z=x+y$
subject to

$$5x+3y\leq 30$$
$$2x+6y\leq 25$$
$$2x-y\leq 8$$
$$x\geq 0, y\geq 0.$$

Then the optimal value of the objective function is

(1) $\dfrac{45}{11}$ (2) $\dfrac{74}{11}$

(3) $\dfrac{85}{12}$ (4) $\dfrac{25}{6}$

Qus 56: Consider a discrete random variable X with the probability mass function

$$P(X=0)=\frac{\theta}{3}, P(X=1)=1-\frac{\theta}{2},$$

$P(X=2)=\dfrac{\theta}{6}$ where $\theta\in(0,1)$ is an unknown parameter. In a random sample of size 90 from this distribution, the observed counts for $X=0,1$ and 2 are 20,60 and 10, respectively. Then, the maximum likelihood estimate of θ is

(1) $\dfrac{1}{3}$ (2) $\dfrac{1}{2}$

(3) $\dfrac{2}{3}$ (4) $\dfrac{3}{4}$

Qus 57: Let $C[0,\pi]$ be the real vector space of real-valued continuous functions on the closed interval $[0,\pi]$. For positive integers n, define $f_n\in C[0,\pi]$ by

$$f_n(x)=\begin{cases}\dfrac{\sin(nx)}{\sin x} & \text{if } x\in(0,\pi),\\ n & \text{if } x=0,\\ (-1)^{n-1}-n & \text{if } x=\pi.\end{cases}$$

Let V be the real subspace of $C[0,\pi]$ spanned by $\{f_1,f_2,f_3\}$. Consider the inner product on V given by

$$\langle f,g\rangle=\frac{2}{\pi}\int_0^{\pi} f(x)g(x)\sin^2 x\,dx$$

Which of the following statements is true?

(1) $f_4\in V$

(2) $\{f_1,f_2,f_3\}$ is an orthonormal basis of V

(3) The dimension of V is 2

(4) $\{f_1,f_2,f_3\}$ is an orthogonal set but not orthonormal

Qus 58: A biased six-faced die is tossed once. Suppose that the probability of any prime number showing up is twice that of any non-prime number showing up. Then the probability that an odd number will show up is

(1) $\dfrac{1}{3}$ (2) $\dfrac{2}{3}$

(4) $\dfrac{4}{9}$ (4) $\dfrac{5}{9}$

Qus 59: Which of the following statements is true?

(1) There exists an entire function f such that $f^{(n)}(0)=\frac{n!}{n^n}$ for all positive integers n

(2) There exists an entire function f such that $f^{(n)}(0)=n!n^n$ for all positive integers n

(3) There exists an entire function f such that $f^{(n)}(0)=(n-1)!$ for all positive integers n

(4) There exists an entire function f such that $f^{(n)}(0)=n!n$ for all positive integers n

Qus 60: Suppose that we have a data set consisting of $2n+1$ observations for some $n\in\mathbb{N}$. Value of each observation is either x or $x+r$, where $x\in\mathbb{N}, r\geq 0$. Then, which of the following statements is always true?

(1) The mean and median of the data will be different if and only if $r>0$

(2) Variance of the data is positive if and only if $r>0$

(3) Mean and mode of the data will be same if and only if $r=0$

(4) Median and mode of the data will be same for all values of $r\geq 0$

Section :- PART - C:-

Qus 61: If the incidence matrix of a block design is given by

$$N=\begin{bmatrix}1&1&1&0\\1&1&0&1\\1&0&1&1\\0&1&1&1\end{bmatrix}$$ then which of the following statements are true?

(1) The design is incomplete

(2) The design is connected

(3) The design is balanced

(4) The design is orthogonal

Qus 62: For a continuous function q defined on $\mathbb{R}$, consider the ordinary differential equation (ODE) $\frac{d^2y}{dx^2}+q(x)y=0,\ x\in\mathbb{R}$.

Then which of the following statements are False?

(1) There exists a q such that $\cos(x)$ and $e^x\cos(x)$ are solutions of ODE

(2) There exists a q such that $\sin(x)$ and $\cos(x)$ are solutions of ODE

(3) There exists a q such that $e^x\sin(x)$ and $e^x\cos(2x)$ are solutions of ODE

(4) There exists a q such that xe^x and $x(x-1)e^x$ are solutions of ODE

Qus 63: A system has two components C_1 and C_2 put in parallel. The components C_1 and C_2 have independent lifetime X_1 and X_2, respectively. The probability distribution of X_j is exponential with mean $\frac{1}{j}, j=1,2$. Suppose that $R(t)$ and $h(t)$ are the reliability and the hazard rate functions of the system, respectively. Then, which of the following statements are true?

(1) $R(t) = e^{-2t} + e^{-t} - e^{-3t}, t > 0$

(2) The expected lifetime of the system is 1

(3) $h(1) = \dfrac{2e + e^2 - 3}{e + e^2 - 1}$

(4) $h(3) = \dfrac{4e^3 + e^6 - 5}{e^3 + e^6 - 1}$

Qus 64: Let X be a random sample of size one from the probability density function

$$f(x \mid \theta) = \begin{cases} \theta e^{-\theta(x-1)}, & \text{if } x > 1 \\ 0, & \text{otherwise} \end{cases},$$

where $\theta(>0)$ is the unknown parameter. Suppose we want to test the null hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta \neq 1$, based on the observed value x of X. Then, which of the following statements are true?

(1) The likelihood function is maximized at $\theta = \dfrac{1}{x-1}$

(2) The maximum value of the likelihood function is $\dfrac{e^{-1}}{x-1}$

(3) The likelihood ratio test for testing H_0 against H_1 rejects H_0 if $(x-1)e^{-x} < k$, for some $k > 0$

(4) The likelihood ratio test for testing H_0 against H_1 rejects H_0 if $x > c$, for some $c > 1$

Qus 65: Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_{10}, Y_{10})$ be a random sample from a bivariate normal distribution BVN$(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ with $\mu_1 = 5$, $\mu_2 = 6$, $\sigma_1^2 = 4, \sigma_2^2 = 9$ and $\rho = \dfrac{1}{2}$. Then which of the following statements are true?

(1) The distribution of $\dfrac{1}{\sqrt{7}} \sum_{i=1}^{10} (X_i - Y_i + 1)$ is $N(0,10)$

(2) The distribution of $\dfrac{1}{19} \sum_{i=1}^{10} (X_i + Y_i - 11)^2$ is χ^2-distribution with degrees of freedom 10

(3) The distribution of $\dfrac{2\sqrt{2}(X_1 - 5)}{\sqrt{\sum_{i=3}^{10} (X_i - 5)^2}}$ is t-distribution with degrees of freedom 9

(4) The distribution of $\dfrac{2\sum_{i=1}^{3} (Y_i - 6)^2}{\sum_{i=4}^{9} (Y_i - 6)^2}$ is F-distribution with degrees of freedom 3 and 6

Qus 66: Consider the $M/M/1$ queue in which customers arrive according to a Poisson process with rate 3 and successive service times are independent exponential random variables having mean $\dfrac{1}{9}$. Let P_n be the long run probability that there are exactly n customers in the system. Then, which of the following statements are true?

(1) $P_0 = \dfrac{1}{3}$

(2) $P_1 = \dfrac{2}{9}$

(3) The average number of customers in the system is 1

(4) The average amount of time that a customer spends in the system is $\dfrac{1}{6}$

Qus 67: Let f be a bounded, twice continuously differentiable real-valued function on $(0, \infty)$ such that $f''(x) \geq 0$ for all $x \in (0, \infty)$. Which of the following statements are true?

(1) $f'(x) \leq 0$ for all $x > 0$

(2) $\lim_{x \to \infty} f'(x) = 0$

(3) $\lim_{x \to \infty} x f'(x)$ need not exist

(4) $\lim_{x \to \infty} x f'(x) = 0$

Qus 68: Let $(f_n)_{n\geq 1}$ be a sequence of real-valued functions on $\mathbb{R}$. Which of the following statements are true?

(1) If each f_n is uniformly continuous and $(f_n)_{n\geq 1}$ converges to f uniformly, then f is uniformly continuous

(2) If each f_n is bounded and $(f_n)_{n\geq 1}$ converges to f pointwise, then f is bounded

(3) If each f_n is bounded and continuous, $(f_n)_{n\geq 1}$ converges pointwise to a bounded and continuous function f, then the convergence is uniform

(4) If each f_n is differentiable and $(f_n)_{n\geq 1}$ converges to f uniformly, then f is differentiable

Qus 69: Let V be the $\mathbb{R}$ - vector space of all polynomials with real coefficients. Let $f(x)=x^2+x+1$. Which of the following subsets of V are linearly independent?

(1) $\{f'(x), f(x)-f(x-1), 1\}$

(2) $\{f(x+1)-f(x), f(x)-f(x-1), 1\}$

(3) $\{f(x), f'(x), 1\}$

(4) $\{f(x+1), f(x-1), f(x)\}$

Qus 70: Consider the sequence $(s_n)_{n\geq 1}$ and $(t_n)_{n\geq 1}$ defined by $s_n=\sum_{k=0}^{n}\frac{1}{(k!)^2}$ and $t_n=\sum_{k=0}^{n}\binom{n}{k}\frac{(-1)^k}{n^k}$. Which of the following statements are true?

(1) $\limsup_{n\to\infty} t_n \leq \limsup_{n\to\infty} s_n$

(2) $\limsup_{n\to\infty} t_n \leq e$

(3) $\liminf_{n\to\infty} s_n \geq e^2$

(4) $\liminf_{n\to\infty} t_n \geq e$

Qus 71: Let f and g be real-valued Riemann integrable functions on $[a,b]$ such that $g([a,b])\subseteq[a,b]$. Which of the following statements are necessarily true?

(1) The composition $f\circ g$ is Riemann integrable

(2) If $g(x)\neq 0$ for each $x\in[a,b]$, then $\frac{f}{g}$ is Riemann integrable

(3) The positive square root $\sqrt{f^2+g^2}$ is Riemann integrable

(4) The composition $f\circ g$ is Riemann integrable, if both f and g are continuous

Qus 72: Let $C[0,1]$ be the $\mathbb{R}$-vector space of real valued continuous functions equipped with the norm $\|f\|=\sup_{x\in[0,1]}|f(x)|$.

Let $T:C[0,1]\to C[0,1]$ be defined as

$T(f)(x)=\int_0^x f(t)\,dt,$ for $x\in[0,1]$

Let $T^n=T\circ T\circ\ldots\circ T\,(n\,times)$. Which of the following statements are true?

(1) There exists $\alpha\in(0,1)$ such that for all $f,g\in C[0,1], \|T(f)-T(g)\|\leq\alpha\|f-g\|$

(2) There exists $\alpha\in(0,1)$ such that for all $f,g\in C[0,1], \|T^2(f)-T^2(g)\|\leq\alpha\|f-g\|$

(3) The set $\{f\in C[0,1]:T(f)=f\}$ is singleton set

(4) $\|T^n\|\to\infty$ as $n\to\infty$

Qus 73: Suppose $u=u(x,y)$ is the solution of the boundary value problem

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial x}=0$ in

$\{(x,y)\in\mathbb{R}^2 : x^2+y^2<1\}$,

$u(x,y)=1+2x^2y^2$ on

$\{(x,y)\in\mathbb{R}^2 : x^2+y^2=1\}$.

Then which of the following statements are true?

(1) The minimum value of u is 1
(2) The maximum value of u is 3
(3) The minimum value of u is 2
(4) The maximum value of u is $\frac{3}{2}$

Qus 74: Let the random vector $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$ have the positive definite dispersion matrix $\begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix}$.

Then which of the following statements are true?

(1) ρ may be -0.47
(2) The first principal component can only explain 32% of the total variation for some ρ
(3) The second principal component can explain more then 32% of the total variation for any ρ
(4) The variance of the first principal component is $1+2\rho$ for any ρ

Qus 75: Consider a Markov chain $\{X_n : n\ge 1\}$ on state space $\{1,2,3,4,5\}$ with the transition probability matrix

$$P\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Then which of the following statements are true?

(1) Stationary distribution is $(0,0,0,0,1)$
(2) State 5 is absorbing and recurrent
(3) All states are aperiodic
(4) $\lim_{n\to\infty} p_{55}^{(n)}=1$

Qus 76: Let $(X_1,X_2,...,X_7)$ and $(Y_1,Y_2,...,Y_9)$ be two independent random samples from the continuous distribution functions $F(x-\mu)$ and $F(x-\theta)$, respectively, where F,μ and θ are all unknown. Further, let μ be the unique median of $F(x-\mu)$ and θ be the unique median of $F(x-\theta)$. Let R_i be the rank of Y_i in the combined sample $i=1,2,...,9$. For testing $H_0:\mu=\theta$ against $H_1:\mu>0$, the statistic $T=\sum_{i=1}^{9}R_i$ is proposed. Then, which of the following statements are true?

(1) The maximum possible value of T is 115
(2) Right-tailed test based on T is appropriate for testing H_0 against H_1
(3) Under $H_0, E(T)=76$
(4) Under $H_0, P(R_1-1, R_9=16)=\frac{1}{240}$

Qus 77: Let $f:\mathbb{Q}[X]\to\mathbb{Q}[X]$ be a ring homomorphism with $f(1)=1$. For $n\ge 1$, let $f^n=\underbrace{f\circ\ldots\circ f}_{n-times}$. Which of the following statements are true?

(1) If f is onto, then so is f^n for all $n\ge 1$
(2) $\ker f^{n+1}=\ker f^n$ for some $n\ge 1$
(3) If f is onto, then f is one-to-one
(4) If f is one-to-one, then f is onto

Qus 78: Let $p:\mathbb{R}\to\mathbb{R}$ be a non-constant polynomial. Which of the following statements are true?

(1) The preimage of a compact set under p is a compact set

(2) The preimage of a connected set under p is a connected set

(3) Every point $x\in\mathbb{R}$ has an open neighbourhood U_x such that the restriction $p\,|\,U_x$ is a homeomorphism onto an open set in $\mathbb{R}$

(4) The image of a bounded set under p is a bounded set

Qus 79: Let G_1 and G_2 be subgroups of a group G. Which of the following statements are true?

(1) If G_1 is normal in G, then $(G_2G_1)/G_1\cong G_2(G_2\cap G_1)$

(2) If H_1 and H_2 are normal subgroups of G_1 and G_2, respectively, then $(G_1\times G_2)/(H_1\times H_2)\cong(G_1/H_1)\times(G_2/H_2)$

(3) If G_1 is normal in G_2 and G_2 is normal in G, then G_1 is normal in G

(4) Every subgroup of prime index in G is normal

Qus 80: Let $f:\mathbb{R}^2\to\mathbb{R}$ be a twice continuously differentiable non-zero function such that $f(tx_1,tx_2)=t^3f(x_1,x_2)$, for all $t>0$ and $(x_1,x_2)\in\mathbb{R}^2$. Which of the following statements are necessarily true?

(1) $3\dfrac{\partial f}{\partial x_1}(1,1)+3\dfrac{\partial f}{\partial x_2}(1,1)=f(1,1)$

(2) $\dfrac{\partial f}{\partial x_1}(1,-1)-\dfrac{\partial f}{\partial x_2}(1,-1)=3f(1,-1)$

(3) $x_1^2\dfrac{\partial^2 f}{\partial x_1^2}(x_1,x_2)+x_2^2\dfrac{\partial^2 f}{\partial x_2^2}(x_1,x_2)+2x_1x_2\dfrac{\partial^2 f}{\partial x_1\partial x_2}(x_1,x_2)=6f(x_1,x_2)$

(4) $x_1^2\dfrac{\partial^2 f}{\partial x_1^2}(x_1,x_2)+x_2^2\dfrac{\partial^2 f}{\partial x_2^2}(x_1,x_2)+2x_1x_2\dfrac{\partial^2 f}{\partial x_1\partial x_2}(x_1,x_2)=9f(x_1,x_2)$

Qus 81: For a group G, let Aut(G) denote the group (under composition) of all bijective group homomorphisms from G onto itself. Which of the following statements are true?

(1) If G_1,G_2 are two groups such that Aut(G_1) is isomorphic to Aut (G_2), then G_1 is isomorphic to G_2

(2) If $|G|=2$, then Aut $(G\times G)$ is abelian

(3) If G is the group of complex numbers under addition, then Aut (G) is abelian

(4) If G is finite, then Aut (G) is finite

Qus 82: Let $f(X)=X^5+X+1\in\mathbb{Q}[X]$ and $g(X)=X^5-X+1\in\mathbb{Q}[X]$. Which of the following statements are true?

(1) $f(X)$ is irreducible in $\mathbb{Q}[X]$, but $g(X)$ is not

(2) $g(X)$ is irreducible in $\mathbb{Q}[X]$, but $f(X)$ is not

(3) Both $f(X)$ and $g(X)$ are irreducible in $\mathbb{Q}[X]$

(4) Neither $f(X)$ nor $g(X)$ is irreducible in $\mathbb{Q}[X]$

Qus 83: Let f be an entire function. Which of the following statements are true?

(1) If $f(z)=f(z+1)$ for all $z\in\mathbb{C}$ then f is a constant function

(2) If $f(z)=f(z+1)=f(z+i)$ for all $z\in\mathbb{C}$ then f is a constant function

(3) If $f\left(\dfrac{1}{z}\right)$ has a removable singularity at 0 then f is a constant function

(4) If f is a non-constant function then $f\left(\frac{1}{z}\right)$ has a pole at 0

Qus 84: Let $u(x)$ be the solution to the Volterra integral equation

$$u(x)=x^2+4\int_0^x (t-x)^2 u(t)\,dt$$

Then which of the following statements are true?

(1) $u(0)=0$

(2) $u\left(\frac{2\pi}{\sqrt{3}}\right)=\frac{1}{6}\left(\frac{4\pi}{e^{\sqrt{3}}-e}-\frac{2\pi}{\sqrt{3}}\right)$

(3) $u\left(\frac{\pi}{2\sqrt{3}}\right)=\frac{1}{6}\left(\frac{\pi}{e^{\sqrt{3}}-\sqrt{3}e}-\frac{\pi}{2\sqrt{3}}\right)$

(4) $u\left(\frac{\pi}{2\sqrt{3}}\right)=\frac{1}{6}\left(\frac{\pi}{e^{\sqrt{3}}+\sqrt{3}e}-\frac{\pi}{\sqrt{3}}\right)$

Qus 85: Let X and Y be independent and identically distributed $N(0,1)$ random variables. Let $S=X^2+Y^2$ and $T=e^{-(X^2+Y^2)/2}$. Then which of the following statements are true?

(1) The probability density function of S is

$$f_S(s)=\begin{cases}\frac{1}{2}e^{-s/2}, & \text{if } s>0\\ 0, & \text{otherwise}\end{cases}$$

(2) The probability density function of T is

$$f_T(t)=\begin{cases}1, & \text{if } 0<t<1\\ 0, & \text{otherwise}\end{cases}$$

(3) $Var(S)=2$

(4) $E(T)=\frac{2}{3}$

Qus 86: Let $M_2(\mathbb{R})$ denote the space of real 2×2 matrices. Let S be the vector subspace of $M_2(\mathbb{R})$ comprising of all symmetric matrices. Let $F:M_2(\mathbb{R})\to S$ be the map defined by $F(X)=XX^T$. Let $DF_A:M_2(\mathbb{R})\to S$ be the derivative of F at $A\in M_2(\mathbb{R})$. Which of the following statements are true?

(1) If $AA^T=I$, then $DF_A:M_2(\mathbb{R})\to S$ is surjective

(2) If $AA^T=I$, then $DF_A:M_2(\mathbb{R})\to S$ need not be surjective

(3) If A is invertible, then $DF_A:M_2(\mathbb{R})\to S$ is surjective

(4) If A is not invertible, then $DF_A:M_2(\mathbb{R})\to S$ is surjective

Qus 87: Let $X_1,X_2,\ldots,X_n\ (n\geq 3)$ be a random sample from the uniform distribution on the interval $(\theta_1-\theta_2,\theta_1+\theta_2)$, where $\theta_1\in\mathbb{R}$ and $\theta_2>0$ are unknown parameter. Let $X_{(j)}$ be the j^{th} order statistic, $j=1,2,\ldots,n$ and let $\bar{X}=\frac{1}{n}\sum_{i=1}^n X_i$, Here $\left(X_{(1)},X_{(n)}\right)$ is a complete and sufficient statistic for (θ_1,θ_2). Then, which of the following statements are true?

(1) $\bar{X}$ is an unbiased estimator of θ_1

(2) $\left(\bar{X}-X_{(1)}\right)$ is an unbiased estimator of θ_2

(3) $\frac{X_{(1)}+X_{(n)}}{2}$ is the uniformly minimum variance unbiased estimator of θ_1

(4) $\frac{(n+1)\left(X_{(n)}-X_{(1)}\right)}{2(n-1)}$ is the uniformly minimum variance unbiased estimator of θ_2

Qus 88: Let $S,T:\mathbb{C}^2\to\mathbb{C}^2$ be $\mathbb{C}$-linear transformations and I denote the identity transformation on $\mathbb{C}^2$. Which of the following statements are necessarily true?

(1) $(ST-TS)^2=\lambda I$ for some $\lambda\in\mathbb{C}$

(2) The characteristic polynomial of $(ST-TS)^2$ is $(x-\lambda)^2$ for some $\lambda \in \mathbb{C}$

(3) If $ST-TS$ has only one eigenvalue, then $ST-TS=\lambda I$ for some $\lambda \in \mathbb{C}$

(4) If $ST-TS$ has only one eigenvalue, then $(ST-TS)^2$ is the zero transformation

Qus 89: Let A,B be distinct 2×2 real matrices. Which of the following statements are true?

(1) If A is invertible, then AB and BA have the same minimal polynomial

(2) If 0 is an eigenvalue of A, then 0 is an eigenvalue of AB

(3) If 0 is the only eigenvalue of A and of B, then 0 is the only eigenvalue of AB

(4) If AB and BA have the same minimal polynomial, then either A or B is invertible

Qus 90: Let f be an entire function which is not a polynomial.

Let $A=\left\{\alpha \in \mathbb{C} \mid f^{(n)}(\alpha) \neq 0 \text{ for all } n \geq 0\right\}$.

Which of the following statements are true?

(1) A is nonempty

(2) A is finite

(3) A is infinite

(4) A is uncountable

Qus 91: Consider the field $\mathbb{F}_3$ consisting of 3 elements. Let V be an $\mathbb{F}_3$-vector space of dimension 3 and W an $\mathbb{F}_3$-vector space of dimension 2. Which of the following statements are true?

(1) The number of two dimensional subspaces of V is 13

(2) The number of surjective linear transformations from V to W is 624

(3) The number of one dimensional subspaces of V is 13

(4) The number of linear transformations from V to W is 3^6

Qus 92: If $\alpha,\beta \in \mathbb{R}$ are such that the equation $\int_0^3 f(x)\,dx=\frac{3}{2}\left[f(\alpha)+f(\alpha+\beta)\right]$ holds for all polynomials $f(x)$ of degree less than or equal to 2, then which of the following statements are true?

(1) $(\alpha,\beta)=\left(\frac{3-\sqrt{3}}{2},\sqrt{3}\right)$ or $(\alpha,\beta)=\left(\frac{3+\sqrt{3}}{2},-\sqrt{3}\right)$

(2) $(\alpha,\beta)=\left(\frac{3-\sqrt{2}}{2},\sqrt{2}\right)$ or $(\alpha,\beta)=\left(\frac{3+\sqrt{2}}{2},-\sqrt{2}\right)$

(3) $(\alpha,\beta)=\left(\frac{3-\sqrt{5}}{2},\sqrt{5}\right)$ or $(\alpha,\beta)=\left(\frac{3+\sqrt{5}}{2},-\sqrt{5}\right)$

(4) $(\alpha,\beta)=\left(\frac{3-\sqrt{7}}{2},\sqrt{7}\right)$ or $(\alpha,\beta)=\left(\frac{3+\sqrt{7}}{2},-\sqrt{7}\right)$

Qus 93: Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with the probability density function

$$f(x\mid\theta)=\begin{cases} e^{-(x-\theta)}, & \text{if } x \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

where $\theta \in \mathbb{R}$ is the unknown parameter. Further, let $Y=\max(X_1, X_2, \ldots, X_n)$. Which of the following are confidence intervals for θ with the confidence coefficient $(1-\alpha)$, where $\alpha \in (0,1)$?

(1) $\left(Y+\frac{1}{n}\ln\left(1-\frac{\alpha}{2}\right), Y-\frac{1}{n}\left(n-\frac{\alpha}{2}\right)\right)$

(2) $\left(Y+\frac{1}{2n}\ln(\alpha), Y\right)$

(3) $\left(Y+\frac{2}{n}\ln\left(\frac{\alpha}{2}\right), Y+\frac{2}{n}\ln\left(1-\frac{\alpha}{2}\right)\right)$

(4) $\left(Y+\frac{1}{n}\ln\left(\frac{\alpha}{2}\right), Y+\frac{1}{n}\ln\left(1-\frac{\alpha}{2}\right)\right)$

Qus 94: Let X be a random variable with the cumulative distribution function

$$F_X(x)=\begin{cases} 0 & , \text{ if } x<0 \\ \frac{x+2}{5} & , \text{ if } 0\le x<2 \\ 1 & , \text{ if } x\ge 2 \end{cases}$$

Then, which of the following statements are true?

(1) $E(X^2)=\frac{4}{3}$

(2) $P\left(\left|X-\frac{4}{5}\right|\ge 1\right)=\frac{19}{25}$

(3) The upper bound of $P\left(\left|X-\frac{4}{5}\right|\ge 1\right)$, using Chebyshev's inequality, is $\frac{52}{75}$

(4) The value of the moment generating function $M_X(t)$, at $t=1$ is $\frac{e^2-1}{5}$

Qus 95: Let G be a group, H is subgroup of G, and $T=\{gH \mid g\in G\}$, the set of all left cosets of H is G. Let S_T be the set of all permutations of T and $\pi: G\to S_T$ be the map defined by $\pi(g)(g_1H)=gg_1H$. For a prime number p, let $\mathbb{F}_p$ denote the field with p elements. In which of the following cases is $\ker\pi$ trivial?

(1) $G=GL_2(\mathbb{F}_p)$ and H is a subgroup of order p

(2) $G=SL_2(\mathbb{F}_p)$ and H is a subgroup of order p

(3) $p\equiv 3(\text{mod } 4), G=GL_2(\mathbb{F}_p)/SL_2(\mathbb{F}_p)$ and H is a subgroup of order 2

(4) $p\equiv 1(\text{mod } 4), G=GL_2(\mathbb{F}_p)/SL_2(\mathbb{F}_p)$ and H is a subgroup of order 2

Qus 96: Consider the initial value problem (IVP)

$$y'+y=0, y(0)=1.$$

Let (y_n) be the iterates of forward Euler method, applied to the IVP, with step size h where $0<h<1$.
Then which of the following statements are true?

(1) The sequence (y_n) does not converge

(2) $y_n\to 0$ as $n\to\infty$

(3) $0\le y_n\le 1$ for $n=0,1,2,...$

(4) $|y(nh)-y_n|\to 0$ as $n\to\infty$

Qus 97: Let X and Y be independent Poisson random variables with means 4 and 2, respectively. Then, which of the following statements are true?

(1) The conditional distribution of X given $X+Y=3$ is Binomial $\left(3,\frac{1}{3}\right)$

(2) $P(X\le 1 \mid X+Y=3)=\frac{7}{27}$

(3) $E(X \mid X+Y=3)=2$

(4) The value of the characteristic function of $X+Y$ at the point $t=\pi$ is e^{-12}

Qus 98: Let $T:\mathbb{C}^7\to\mathbb{C}^7$ be $\mathbb{C}$-linear operator with eigenvalues 2,3 and 5. Consider the subspace $W:=\{v\in\mathbb{C}^7 : (T-5I)^k v=0$ for some integer $k>0\}$ of $\mathbb{C}^7$. Suppose that $(T-2I)^2(T-3I)^2(T-5I)^2=0$. Which of the of the following statements are necessarily true?

(1) T has at least four linearly independent eigenvectors

(2) $\dim W \geq 2$

(3) $\ker\left((T-2I)^{2025}\right)=\ker\left((T-2I)^{2026}\right)$

(4) $(T-2I)(T-3I)$ is a nilpotent operator

Qus 99: Which of the following matrices are similar over $\mathbb{R}$ to the matrix

$$A=\begin{pmatrix}-1 & 1 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 0 & 1\end{pmatrix}?$$

(1) $\begin{pmatrix}0 & 0 & 0 & -1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 2\\ 0 & 0 & 1 & 0\end{pmatrix}$

(2) $\begin{pmatrix}0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & 0\end{pmatrix}$

(3) $\begin{pmatrix}0 & 1 & 1 & 0\\ 1 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

(4) $\begin{pmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

Qus 100: For each $n \geq 1$, let $f_n:\mathbb{R}\to\mathbb{R}$ be a function defined by $f_n(x)=\dfrac{e^{-n^2x^2}}{n}$.

Which of the following statements are true?

(1) $(f_n)_{n\geq 1}$ converges uniformly to 0 on $\mathbb{R}$, and $(f_n')_{n\geq 1}$ converges uniformly to 0 on the interval $(-M,M)$ for some positive real number M

(2) $(f_n)_{n\geq 1}$ converges uniformly to 0 on $\mathbb{R}$, and $(f_n')_{n\geq 1}$ converges pointwise to 0 on $\mathbb{R}$

(3) $(f_n)_{n\geq 1}$ converges uniformly to 0 on $\mathbb{R}$ and $(f_n')_{n\geq 1}$ does not converge pointwise to 0 on $\mathbb{R}$

(4) $(f_n)_{n\geq 1}$ converges pointwise to 0 on $\mathbb{R}$ but not uniformly on $\mathbb{R}$

Qus 101: Let V be a finite dimensional complex inner product space. For a linear map $T:V\to V$, let T^* denote its adjoint. Which of the following statements are true?

(1) If trace of TT^* is zero,then $T=0$

(2) Let $v\in V$ be such that $T^*T(v)=0$. Then $T(v)=0$

(3) Suppose $T=T^*$ and $N>1$ be an integer. Let $v\in V$ be such that $T^{2^N}(v)=0$. Then $T(v)=0$

(4) Suppose that $TT^*=T^*T$ and $N>1$ be an integer. Let $v\in V$ be such that $T^N(v)=0$. Then $T(v)=0$

Qus 102: Let $X_1,X_2,...,X_n$ be a random sample from $N(\theta,1),\theta\in\mathbb{R}$. If $\hat{\theta}$ is the Bayes estimator of θ with respect to some prior $\pi(\theta)$ and loss function $L(\theta,d)$. Then, which of the following statements are true?

(1) $\hat{\theta}=\dfrac{\sum_{i=1}^{n}X_i}{n+T^2}$, if the prior is $N\left(0,\dfrac{1}{T^2}\right),T^2$ known and $L(\theta,d)=(\theta-d)^2$

(2) $\hat{\theta}=\dfrac{\sum_{i=1}^{n}X_i}{n+T^2}$, if the prior is $N\left(0,\dfrac{1}{T^2}\right),T^2$

known and $L(\theta,d)=|\theta-d|$

(3) $\hat{\theta}=\dfrac{\sum_{i=1}^{n}X_i}{n+\frac{1}{T^2}}$, if the prior is $N\left(0,\frac{1}{T^2}\right), T^2$

known and $L(\theta,d)=|\theta-d|$

(4) $\hat{\theta}=\dfrac{\sum_{i=1}^{n}X_i}{n}$, if the prior is the Jeffreys prior

and $L(\theta,d)=(\theta-d)^2$

Qus 103: For $a,b,c\in\mathbb{R}$, consider the variational problem

Minimize $J[y]=\int_0^2\left[a(y')^2+2byy'+cy^2\right]dx$

subject to

$$y(0)=10, y(1)=100$$

Then which of the following statements are true?

(1) If $(a,b,c)=(-2,1,-2)$, then every adimissible extremal is a minimizer

(2) If $(a,b,c)=(1,0,2)$, then every admissible extremal is a minimizer

(3) If $(a,b,c)=(2,-1,1)$, then every admissible extremal is a minimizer

(4) If $(a,b,c)=(1,-2,5)$, then every admissible extremal is a minimizer

Qus 104: Consider a paired data

$(x_i,y_i):i=1,2,3,4,5$, where

$(x_1,x_2,x_3,x_4,x_5)=(-2,-1,0,1,2)$ and

$y_i=x_i^2$ for all $i=1,2,3,4,5$. On this data, a simple linear regression model with an intercept term and a simple linear resgression model without an intercept term are fitted using the method of least squares. Which of the following statements are true?

(1) The two fitted lines have the same slope

(2) The two fitted models have the same intercept

(3) The model with intercept passes through at least one of the observed data points

(4) The model without intercept passes through at least one of the observed data points

Qus 105: Let $\mathbb{D}^\times=\{z\in\mathbb{C}:0<|z|<1\}$ be the punctured unit disk the f be a bijective holomorphic map of $\mathbb{D}^\times$ onto itself. Which of the following statements are true?

(1) $\lim_{z\to 0}f(z)$ does not exist

(2) $\lim_{z\to 0}f(z)$ exists and has absolute value ≤ 1

(3) $\lim_{z\to 0}f(z)=0$

(4) There exists $\theta\in\mathbb{R}$ such that $f(z)=e^{i\theta}z$ for all $z\in\mathbb{D}^\times$

Qus 106: Let $p>2$ be a prime number. Let $\mathbb{F}_p$ denote the field with p elements and $\overline{\mathbb{F}}_p$ an algebraic closure of $\mathbb{F}_p$. Which of the following statements are true?

(1) Let $f(X)\in\mathbb{F}_p[X]$ and α be a root of f in $\overline{\mathbb{F}}_p$. Then $\mathbb{F}_p(\alpha)$ is the splitting field of f in $\overline{\mathbb{F}}_p$

(2) Let $f,g\in\mathbb{F}_p[X]$ be irreducible polynomials of same degree and α be a root of f in $\overline{\mathbb{F}}_p$. Then $\mathbb{F}_p(\alpha)$ is the splitting field of g in $\overline{\mathbb{F}}_p$

(3) $\mathbb{F}_p[X]$ has infinitely many irreducible polynomials

(4) The set $\{a+b \mid a,b\in\mathbb{F}_p\}$ is contained in $\{a^2+b^2 \mid a,b\in\mathbb{F}_p\}$

Qus 107: For $\alpha\geq 0$, consider the functional

$$J_\alpha[y]=\int_1^2\frac{(y')^2}{x^\alpha}dx$$

defined for all continuously differentiable functions defined on the interval $[1,2]$ satisfying the conditions $y(1)=1, y(2)=2$.

Then which of the following statements are true?

(1) $y(x)=\frac{1}{15}(x^4+14)$ is an extremal for J_3

(2) $y(x)=\frac{1}{3}(x^2+2)$ is an extremal for J_1

(3) $y(x)=x$ is an extremal for J_0

(4) $y(x)=\frac{1}{2}(x^2-x+2)$ is an extremal for J_1

Qus 108: Let $X_1, X_2, ..., X_n\,(n\geq 2)$ be a random sample from a continuous distribution with the probability density function

$f(x\mid\theta)=\frac{1}{2\theta}e^{-\frac{|x-\theta|}{\theta}}, x\in\mathbb{R}$, where $\theta(>0)$ is an unknown parameter. Let

$U_n=\frac{1}{n}\sum_{i=1}^{n}X_i, V_n=\frac{1}{n}\sum_{i=1}^{n}X_i^2$ and

$S_n^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-U_n)^2$. Then, which of the following statements are true?

(1) U_n is an unbiased estimator of θ

(2) S_n^2 is an unbiased estimator of θ^2

(3) $\frac{1}{3}V_n$ is a consistent estimator of θ^2

(4) The statistic (U_n, V_n) is complete

Qus 109: An analyst fits a multiple linear regression model $Y_i=\beta_0+\beta_1X_{1i}+...+\beta_4X_{4i}+\in_i$, $i=1,2,...,n$, using the method of least squares. However, his coordinator insists that the intercept and two regressors $Z_1=X_1+X_3$ and $Z_2=X_2-X_4$ are enough to represent the model. Suppose the coordinator's claim can be tested in the form of a general linear hypothesis, viz,. $H_0: L^T\beta=0$ against $H_A=H_0$ is not true, where $\beta^T=(\beta_0,\beta_1,...,\beta_4)$. Suppose we have n observations on the response Y and each regressor.

Further assume that the errors with or without restrictions are indepenent $N(0,\sigma^2)$ variables. Then, which of the following statements are true?

(1) A possible choice of L is

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(2) The test statistic for testing H_0 against H_A follows an F-distribution with $(2, n-4)$ degree of freedom under H_0

(3) Sum of the square residuals under the restrictions $L^T\beta=0$ follows $\sigma^2\chi^2_{n-3}$ distribution

(4) Sum of squares residuals without the restrictions $L^T\beta=0$ follows a non-central $\sigma^2\chi^2_{n-4}$ distribution

Qus 110: Let f and K be such that the solution of the initial value problem

$y''+3y'+2y=4\sin(x), y(0)=1, y'(0)=-2$

satisfies the Volterra integral equation

$y(x)=f(x)+\int_0^x K(x,t)y(t)\,dt$.

Then which of the following statements are true?

(1) $f'(\pi)=3$

(2) $f(\pi)+f'(\pi)=4-\pi$

(3) $f(\pi)+f'(\pi)=2-\pi$

(4) $f(0)+f'(0)=-4$

Qus 111: Let $B(v,w)$ be a non degenerate symmetric bilinear form on $\mathbb{R}^2$ and let $q(v)=B(v,v)$ be the corresponding quadratic form. Suppose there exist vectors $v,w\in\mathbb{R}^2$ such that $B(v,v)=0$ and

$B(v,w) \neq 0$. Which of the following statements are necessarily true?

(1) $B(w,w)=0$

(2) There exists an $\alpha \in \mathbb{R}$ such that $q(\alpha v+w)=0$

(3) There are infintely many $\alpha \in \mathbb{R}$ such that $q(\alpha v+w)=0$

(4) q is equivalent to the quadratic form $Q(x,y)=x^2-y^2$ for all $(x,y)\in \mathbb{R}^2$

Qus 112: Consider the Cauchy problem (CP)

$$x\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=1$$

$$u(0,y)=e^y, y\in \mathbb{R}$$

Then which of the following statement are true?

(1) There is no neighbourhood of the origin on which (CP) has a solution

(2) Cauchy problem has a unique solution defined on some neighbourhood of the origin

(3) Cauchy problem has a unique solution defined on some neighbourhood of the point $(0,1)$ in the xy-plane

(4) Cauchy problem has an infinite number of solutions, each of which is defined on some neighbourhood of the origin

Qus 113: Let μ denote the Lebesgue measure on $\mathbb{R}$. Suppose that f is a non-negative Lebesgue measurable function on $\mathbb{R}$. Let $0=a_0<a_1<a_2<...$ be an unbounded sequence such that $a_{n+1}\le ca_n$ for some real number c and for all $n\ge 1$. Let $A_k=\{x\in \mathbb{R} \mid a_k \le f(x)<a_{k+1}\}$ for each $k\ge 0$. Which of the following statements are true?

(1) If f is Lebesgue intergrable on $\mathbb{R}$, then $\sum_{k\ge 0} a_k\mu(A_k)$ is finite

(2) If $\sum_{k\ge 0} a_k\mu(A_k)$ is finite, then f is Lebesgue integrable on $\mathbb{R}$

(3) If $\sum_{k\ge 0} a_k\mu(A_k)$ is finite, and $f(x)\ge a_1$ for all $x\in\mathbb{R}$, then f is Lebesgue integrable on $\mathbb{R}$

(4) If $\sum_{k\ge 0} a_k\mu(A_k)$ is finite and f is bounded, then f is Lebesgue integrable on $\mathbb{R}$

Qus 114: Consider $\mathbb{R}$ with the usual topology and $S=\{a+b\sqrt{2} \mid a,b\in\mathbb{Q}\}\subset\mathbb{R}$ with the subspace topology. Which of the following statements are true?

(1) S is dense in $\mathbb{R}$

(2) $S\setminus\mathbb{Q}$ is dense in $\mathbb{R}$

(3) $S\setminus\mathbb{Q}$ is discrete with subspace topology on S

(4) S is connected

Qus 115: What is the value of the limit

$$\lim_{n\to\infty}\frac{1}{n}\left[(n+1)(n+2)...(n+n)\right]^{1/n} ?$$

(1) $\frac{2}{e}$

(2) $\frac{4}{e}$

(3) $\log_e 2-1$

(4) $2\log_e 2-1$

Qus 116: Let $D=\{(x,y)\in\mathbb{R}^2 : -1\le x\le 1, -1\le y\le 1\}$ and $f:D\to\mathbb{R}$ be the function defined by $f(x,y)=1+\sqrt{y_+}$, where $y_+=\max\{y,0\}$. Consider the initial value problem (IVP) $\frac{dy}{dx}=f(x,y), y(0)=0$.

Then which of the following statement are true?

(1) f is a Lipschitz continuous function on D

(2) f is not a Lipschitz continuous function on D

(3) IVP has at least one solution

(4) IVP has no solution

Qus 117: Suppose f,g are smooth functions of generalized coordinates $q_1,q_2,...,q_n$, the associated conjugate momenta $p_1,p_2,...,p_n$,

and time t. Let $[f,g]$ denote the Poisson bracket of f and g. Suppose H is a Hamiltonian of the system.
Then which of the following statements are true?

(1) $\frac{\partial}{\partial t}[f,g]=\left[\frac{\partial f}{\partial t},g\right]+\left[f,\frac{\partial g}{\partial t}\right]$

(2) If f is a constant of motion, and f is independent of t, then $[H,f]$ is a constant of motion

(3) $[[H,f],g]+[[g,H],f]+[[f,g],H]=0$

(4) If f and g are constants of motion, then $[f,g]$ is constant of motion

Qus 118: Consider the ordinary differential equation (ODE)

$$\frac{d^2y}{dx^2}+(\cos(x))\frac{dy}{dx}+(\sin(x))y=0$$

Let $\varphi_1(x),\varphi_2(x)$ be solution of the ODE, satisfying $\varphi_1(0)=1,\frac{d\varphi_1}{dx}(0)=0$ and

$\varphi_2(0)=0,\frac{d\varphi_2}{dx}(0)=1$

Then which of the following statements are true?

(1) $\varphi_1(x+2\pi)$ is also a solution of ODE

(2) $\varphi_2(x+4\pi)$ is also a solution of ODE

(3) There are no constants a,b such that $\varphi_2(x+4\pi)=a\varphi_1(x)+b\varphi_2(x)$

(4) There exist $a,b\in\mathbb{R}$ such that $\varphi_1(x+2\pi)=a\varphi_1(x)+b\varphi_2(x)$

Qus 119: Let $\gamma:[0,1]\to\mathbb{C}$ be the function $t\mapsto e^{2\pi it}$

and $I=\int_\gamma e^z e^{\frac{1}{z}}dz$.

Which of the following statements are true?

(1) $I=0$

(2) $\frac{1}{2\pi i}I\in\{4n:n\in\mathbb{Z},n\geq 1\}$

(3) $I=2\pi i\sum_{n=0}^{\infty}\frac{1}{n!}$

(4) $I=2\pi i\sum_{n=0}^{\infty}\frac{1}{n!(n+1)!}$

Qus 120: Let $X_1,X_2,\ldots$ be a sequence of independent and identically distributed random vari ables with $E(X_1)=0,E(X_1^2)=1$,

$E(X_1^3)=0$, $E(X_1^4)=3$. Let

$S_n=\sum_{i=1}^n X_i,T_n=\sum_{i=1}^n X_i^2,U_n=\sum_{i=1}^n X_i^3$ and

$V_n=\sum_{i=1}^n X_i^4$. Then, which of the following statements are true?

(1) $\frac{S_n}{\sqrt{n}}$ converges in distribution to a random variable Z, where $Z\sim N(0,1)$

(2) $\frac{T_n-n}{\sqrt{3n}}$ converges is distribution to a random variable Z, where $Z\sim N(0,1)$

(3) $\frac{\sqrt{n}S_n}{T_n}$ converges in distribution to a random variable Z, where $Z\sim N(0,1)$

(4) $\frac{T_n-n}{\sqrt{V_n}}$ converges in distribution to a random variable Z, where $Z\sim N(0,1)$

CSIR MATH SOLUTION

28-July-2025

PART "A"

Q 1. Ans (2)

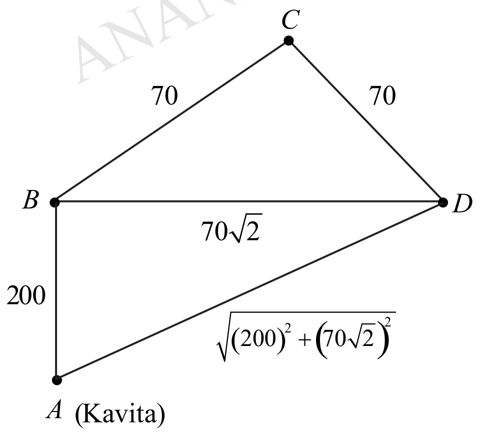
3 friends are Rahman, George and Vedant and colour of their shirts are Red, Green and Violet.

As name & colour of shirt are words starting from different letters, so followings are the possibilities according to given information.

Name	Colour
Rahman	Violet (X), Red (X), Green
Vedant	Red
George	Violet

So, option (2) is correct.

Q 2. Ans (4)



From the above figure Kavita started her journey from A and ended at D. The required distance AD is equal to

$$\sqrt{(200)^2 + \left(70\sqrt{2}\right)^2} =$$

$$\sqrt{40000 + 9800} = \sqrt{49800}$$

```
   2  | 4 9 8 0 0 | 2 2 3. 1...
   2  | 4
  42  |   98
   2  |   84
 443  |   14 0 0
   3  |   13 2 9
4461  |      7 1 0 0
   1  |      4 4 6 1
4462  |      2 6 3 9
```

Which is approximately 223.1 which is close to 223.

Q 3. Ans (4)

Let x be the investment of the trader then on first day there was loss of $\frac{2}{3}x$ and she is left with $x - \frac{2x}{3} = \frac{x}{3}$. Next day she gained $\frac{1}{3} \times \frac{2x}{3} = \frac{2x}{9}$

So, now total she has $\frac{x}{3} + \frac{2x}{9} = \frac{5x}{9}$ amount of wealth left with, so fraction of amount she is left with is $5/9$.

Q 4. Ans (2)

Rakesh owned $\frac{2}{15}$ part of shares of the company and sold it's $\frac{1}{3}$ rd part, so $\frac{2}{15} \times \frac{1}{3} = \frac{2}{45}$ part of shares of the company costs Rs. 75000/, hence total value of the company will be $75000 \times \frac{45}{2} = 16,87,500$ Rs.

Q 5. Ans (1)

Let the number be x. It was to be multiplied by 2, so it's value would have been $2x$. It was wrongly evaluated as $\frac{x}{2}$. Hence change in calculator $=2x-\frac{x}{2}=\frac{3x}{2}$.

So percentege change

$$=\frac{\frac{3x}{2}}{2x}\times 100\%=\frac{3}{4}\times 100\%=75\%$$

Q 6. Ans (2)

Let x be there in the proportion $8:5:7$ then initially in the garden there was $8x$ Roses, $5x$ Lotus & $7x$ marigold.

Now they are increased by 75%, 40% & 50% respectively, so now their amount will be

$8x\left(1+\frac{75}{100}\right),5x\left(1+\frac{40}{100}\right)\&\,7x\left(1+\frac{50}{100}\right)$ i.e.

$14x,7x\,\&\,\frac{21x}{2}$, so their new ratio is

$14:7:\frac{21}{2}::2:1:\frac{3}{2}::4:2:3$

Q 7. Ans (3)

The father of person in photo was son of father of Ramesh, who has no brother & sister, so father of person in photo was Ramesh.

Hence the person in the photo was son of Ramesh.

Q 8. Ans (1)

Given two digit number is ab & if we subtract sum of it's digit from it then we get x, where $x=10\,a+b-(a+b)=9a$ and if we divide x by 9 then we get 'a' always.

Q 9. Ans (4)

Some wooden objects are furnitures.

All chairs are furnitures.

Seats are features of many objects including chairs.

Q 10. Ans (1)

DELTOID is coded as 3152893

$\Rightarrow$

D	E	L	T	O	I	D
3	1	5	2	8	9	3

$\Rightarrow$ $D\equiv 3,\ E\equiv 1,\ L\equiv 5,\ T\equiv 2, O\equiv 8$,
$I\equiv 9\ \&\ D\equiv 3$

So, LOTION will be coded as

58298_ ; where _ cannot be 1

So, only one option is possible $\underline{582986}$.

Q 11. Ans (1)

Volume of water in Cylinder

$=\pi r^2h=\pi(20)^2(25)=10000\pi(cm)^3$

Volume of spherical ball of radius 7cm

$=\frac{4}{3}\pi r^3=\frac{4}{3}\pi(7)^3=\frac{1372\pi}{3}$

Volume of water in cylinder + Volume of spherical ball =

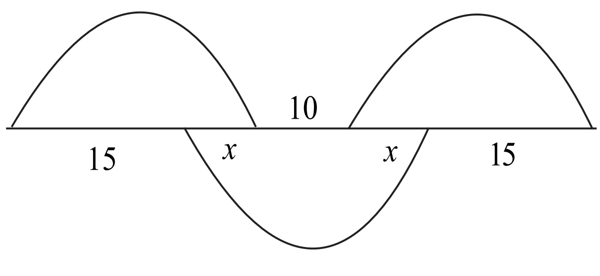
$$\pi\left(10000+\frac{1372}{3}\right)=\left(\frac{31372}{3}\right)\pi\,cm^3$$

Let h be the increment of height then

$$\pi(20)^2(25+h)=\left(\frac{31372}{3}\right)\pi$$

$\Rightarrow$ $400h=\frac{1372}{3}\Rightarrow h=\frac{1372}{1200}=1.14$ (approx)

Q 12. Ans (2)



Diameter of the semicircle in above figure is equal to $d=15+x=10+2x$

$\Rightarrow$ $15+x=10+2x\Rightarrow x=5\Rightarrow d=15+5=20$

Q 13. Ans (3)

When the figure will be squeezed then it will become rectangle of side 2×1 i.e. diameter * radius, so the area of the shape will be 2.

Q 14. Ans (1)

Diameter of the wheel is 36cm, so perimeter of the wheel will be $\pi \times 36\,\text{cm} = 36\pi\text{cm}$

Speed of the wheel $= 60\,\text{km/hr}$

$$= \frac{60\times1000}{3600}\text{m/sec} = \frac{100}{6}\text{m/sec} = \frac{1000}{6}\text{cm/sec}$$

Number of rotation per second

$$= \frac{\frac{10000}{6}}{36\pi} = \frac{1000}{216\pi} = \frac{1250}{27\pi}$$

So, number of rotation per minute

$$= \frac{1250\times60}{27\pi} = \frac{25000}{9\pi} = \frac{25000\times7}{9\times22} = \frac{87500}{99}$$

Which is approximately 884.

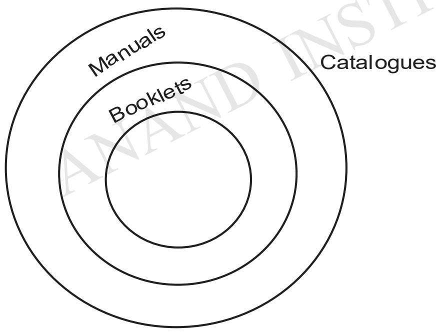
Q 15. Ans (2)

$$1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3 =$$

$$\left(\frac{9\times10}{2}\right)^2 = (45)^2 = 2025$$

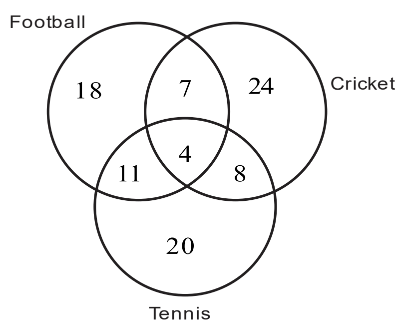
So, it's last digit is 5.

Q 16. Ans (3)



Venn diagram representation of statements I & II from Venn diagram we can conclude that all booklets are catalogues.

Q 17. Ans (3)



Total number of players

$= 18+7+4+11+24+8+20 = 92$

Number of players who play exactly 2 sports

$= 7+8+11 = 26$

So, $\%$ of players who play exactly 2 sports

$= \frac{26}{92}\times100\% = \frac{650}{23}\% = 28.26\%$ which is closest to 28%.

Q 18. Ans (3)

Profit ratio share of the companies are as follows.

$$P = \frac{17}{80} = 0.2125$$

$$Q = \frac{25}{120} = 0.2083$$

$$R = \frac{24}{110} = 0.2181$$

$$S = \frac{14}{80} = 0.175$$

$$T = \frac{20}{100} = 0.2$$

X is false as no two companies have same profit.
Y is true as R has maximum profit.
Z is true as S has minimum profit.

Q 19. Ans (2)

Ratio of initial monthy salaries of John, Riya and Sunil were in proportion 4:3:5.

So, if constant of proportionality is x then salary of John, Riya and Sunil will be $4x, 3x \,\&\, 5x$.

After increment of Rs. 10000/- their salary will be $4x+10000$, $3x+10000$ & $5x+10000$ respectively which are in the ratio 6:5:7.

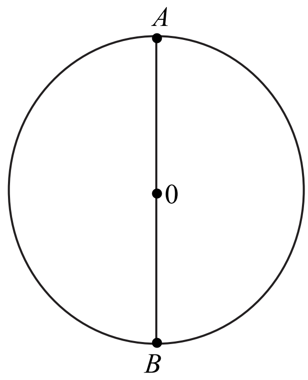
So,

$$\frac{4x+10000}{3x+10000} = \frac{6}{5} \Rightarrow 20x+50000 = 18x+60000$$

$\Rightarrow \quad 2x = 10000 \quad \Rightarrow x = 5000$.

So, salary of Sunil initially was $5x = 5\times5000 = 25000$

Q 20. Ans (2)



Rahul took one round and stopped, so distance covered by Rahul is circumference of the circle $= 2\pi r$.

Distance covered by father of Rahul will be half of perimeter of circle plus diameter of circle $= \pi r + 2r$.

So, ratio of distance of Rahul and his father will be $\dfrac{2\pi r}{\pi r + 2r} = \dfrac{2\pi}{\pi + 2} = \underline{2\pi/(\pi+2)}$.

PART "B"

Q 21. Ans (3)

$$f_n(x) = nx \quad \text{if } x \in \left[0, \frac{1}{n}\right]$$
$$2 - nx \quad \text{if } x \in \left(\frac{1}{n}, \frac{2}{n}\right]$$
$$0 \quad \text{if } x \in \left(\frac{2}{n}, 1\right]$$

$\Rightarrow$ Pointwise limit function is

$$f(x) = \lim_{n\to\infty} f_n(x) = 0\ ;\ \forall x \in [0,1]$$

$\Rightarrow$ $(f_n)_{n\geq 1}$ converges pointwise on $[0,1]$ to a continuous function f.

Now

$$|f_n(x) - f(x)| = |f_n(x) - 0| = |f_n(x)| = g(x)$$
$$= nx\ ;\ x \in \left[0, \frac{1}{n}\right]$$
$$= 2 - nx\ ;\ x \in \left(\frac{1}{n}, \frac{2}{n}\right]$$
$$= 0\ ;\ x \in \left[\frac{2}{n}, 1\right]$$

$$g'(x) = n\ ;\ x \in \left(0, \frac{1}{n}\right)$$
$\Rightarrow$
$$= -n\ ;\ x \in \left(\frac{1}{n}, \frac{2}{n}\right)$$
$$= 0\ ;\ x \in \left(\frac{2}{n}, 1\right)$$

So, $M_n = \sup\limits_{x\in[0,1]} |f_n(x) - f(x)|$

$$= \max\left(n \times \frac{1}{n}, 2 - n \times \frac{1}{n}\right) = \max(1,1)$$
$$= 1$$

$\Rightarrow$ $\lim\limits_{n\to\infty} M_n = \lim\limits_{n\to\infty} 1 = 1 \neq 0$

So, covergence is not uniform.

So, option (3) is correct.

Q 22. Ans (2)

f is entire function such that $f(c)\subset\{x+iy \mid y=x+1\}$ so range of f is contained in a straight line, so by application of Liouville's theorem or by Picard's theorem , $f(z)$ is constant function.

So, $\lim_{|z|\to\infty}\frac{f(z)}{z}=0$

So, option (2) is correct.

Q 23. Ans (2)

Number of invertible matrices of order 2 over z_5 field = Number of matrices of order 2 over z_5 field of rank 2

$=\left(5^2-1\right)\left(5^2-5\right)=24\times 20=480$

Number of matrices of order 2 over z_5 field of rank 0 is 1, as rank of null matrix only is 0.

Total number of matrices of order 2 over z_5 field is $5^{2\times 2}=5^4=625$.

As matrices of order 2 are of rank 0,1 or 2, so number of matrices of order 2 and rank 1 over z_5 field is $625-(480+1)=144$

So, option (2) is correct.

Q 24. Ans (4)

$\gamma_R:[0,1]\to C$ be the map such that $t\mapsto \mathrm{Re}^{2\pi it}$

$\Rightarrow$ $\gamma_R(t)=\mathrm{Re}^{2\pi it}\Rightarrow|\gamma_R(t)|=\left|\mathrm{Re}^{2\pi it}\right|=R$

$\Rightarrow$ $\gamma_R'(t)=2\pi i\,\mathrm{Re}^{2\pi it}$

If $f(z)=a_0+a_1z+a_2z^2+\ldots+a_nz^n\,;a_n\neq 0$

then for $(f\circ\gamma_R)(t)\gamma_R'(t)$

We know as $R\to\infty$ for polynomial of degree n, only dominating term a_nz^n will be used.

So, for $\lim_{R\to\infty}\left|(f\circ\gamma_R)(t)\gamma_R'(t)\right|$ we get

$\left|(f\circ\gamma_R)(t)\gamma_R'(t)\right|\approx\left|a_n\left(\mathrm{Re}^{2\pi it}\right)^n\right|\left|2\pi i\,\mathrm{Re}^{2\pi it}\right|$

$=|a_n|\,2\pi R^{n+1}$

So, $\int_0^1\left|(f\circ\gamma_R)(t)\gamma_R'(t)\right|dt\approx|a_n|R^{n+1}.2\pi$

$\lim_{R\to\infty}\int_0^1\left|(f\circ\gamma_R)(t)\gamma_R'(t)\right|dt=\lim_{R\to\infty}2\pi|a_n|R^{n+1}$

exist iff $n+1<0$ i.e. $n<-1$ (impossible)

or $|a_n|=0;\ \forall n$. i.e. $a_0=a_1=\ldots=a_n=0$

So, $f(z)=0\ ;\ \forall z\in c$

(1) Now $zf\left(\frac{1}{z}\right)=z.0=0$

$\Rightarrow$ $\lim_{|z|\to\infty}zf\left(\frac{1}{z}\right)=\lim_{|z|\to\infty}0=0$

option (1) is true. (Do not tick it)

(2) The function f is $f(z)=0$ which is constant function.

option (2) is true (Do not tick it)

(3) $\lim_{R\to\infty}\int_0^1\left|(f\circ\gamma_R)(t).\gamma_R'(t)\right|dt=\lim_{R\to\infty}0=0=c$

So, option (3) is true (Do not tick it)

(4) $c>0$ is false (so tick it)

Q 25. Ans (2)

$\frac{d}{dx}\left(x\frac{dy}{dx}\right)+\frac{\lambda}{x}y=0$

$\Rightarrow$ $x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{\lambda}{x}y=0$

$\Rightarrow$ $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\lambda y=0$ (1)

Let $x=e^z\Rightarrow z=\ln x$ & $D=\frac{d}{dz}$ then (1) become.

$\left(D(D-1)+D+\lambda\right)y=0$

$\Rightarrow$ $\left(D^2+\lambda\right)y=0$ (2)

Case I:- $\lambda=0$

$\Rightarrow$ $D^2y=0\Rightarrow y=c_1+c_2z\Rightarrow y=c_1+c_2\ln x$

$y(1)=0\Rightarrow c_1=0,\ y\left(e^{2\pi}\right)=0\Rightarrow c_2.2\pi=0$

$\Rightarrow c_2=0$

$\Rightarrow$ $c_1=c_2=0$ $\Rightarrow y=0$

It has trivial solution.

Case II:- $\lambda < 0$; take $\lambda = -k^2$; $k \neq 0$

$\Rightarrow$ $\left(D^2 - k^2\right) y = 0 \Rightarrow y = c_1 e^{kz} + c_2 e^{-kz}$

$\Rightarrow$ $y = c_1 x^k + c_2 x^{-k}$

$y(1) = 0 \Rightarrow c_1 + c_2 = 0$

$y\left(e^{2\pi}\right) = 0 \Rightarrow c_1 e^{2\pi k} + c_2 e^{-2\pi k} = 0$

So, $c_1 = c_2 = 0 \Rightarrow y = 0$

So, it has trivial solution only.

Case III:- $\lambda > 0$; take $\lambda = k^2 ; k \neq 0$

$\Rightarrow$ $\left(D^2 + k^2\right) y = 0 \Rightarrow y = c_1 \cos kz + c_2 \sin kz$

$\Rightarrow$ $y = c_1 \cos k \log x + c_2 \sin k \log x$

$y(1) = 0 \Rightarrow c_1 = 0$ &

$y\left(e^{2\pi}\right) = 0 \Rightarrow c_2 \sin 2\pi k = 0$

$\Rightarrow$ $c_2 = 0$ (not possible) or $\sin 2\pi k = 0$ (possible)

So $2\pi k = n\pi$; $n \in I \Rightarrow k = \frac{n}{2}$; $n \in I \setminus \{0\}$

$\Rightarrow$ $\lambda = k^2 = \frac{n^2}{4}$

$\Rightarrow$ $\lambda_n = \frac{n^2}{4} \Rightarrow \frac{1}{\lambda_n} = \frac{4}{n^2} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{\lambda_n} = \sum_{n=1}^{\infty} \frac{4}{n^2}$

$= 4\sum_{n=1}^{\infty} \frac{1}{n^2} = 4 \times \frac{\pi^2}{6} = \frac{2\pi^2}{3}$

Q 26. Ans (4)

It is Fredholm integral equation

Given $y(x) = x^2 + 2\int_0^1 xt\, y(t)\, dt$

$= x^2 + 2x\int_0^1 t\, y(t)\, dt$

Let $c = \int_0^1 t\, y(t)\, dt$ (1)

putting in (1) we get

$c = \int_0^1 t\left(t^2 + 2ct\right) dt = \frac{1}{4} + \frac{2c}{3} \Rightarrow \frac{c}{3} = \frac{1}{4}$

$\Rightarrow$ $c = \frac{3}{4}$

$\Rightarrow$ $y(x) = x^2 + 2x\left(\frac{3}{4}\right) \Rightarrow y(x) = x^2 + \frac{3x}{2}$

$\Rightarrow$ $y'(x) = 2x + \frac{3}{2}$

$\Rightarrow$ $y(0) = 0$; $y(1) = \frac{5}{2}, y(-1) = -\frac{1}{2}, y'(0) = \frac{3}{2}$

$y'(1) = \frac{7}{2}$, $y'(-1) = -\frac{1}{2}$

So, $y'(-1) + y'(1) = -\frac{1}{2} + \frac{7}{2} = 3$.

So, option (4) is correct.

Q 27. Ans (2)

(1) For any prime $p, (p-1)! \equiv -1 (\text{mod } p)$
(By Wilson Theorem)

$\Rightarrow$ $p / (p-1)! + 1$

(2) For any prime p, and $a \in \mathbb{Z}$
(By Fermat's Theorem)

$a^{p-1} \equiv 1 (\text{mod } p)$

$\Rightarrow$ $p \mid a^{p-1} - 1$

$\Rightarrow$ $p \mid (1234)^{p-1} - 1$

(3) Also, $a^{p-1} \equiv 1 (\text{mod } p)$

$\Rightarrow$ $a^p \equiv a (\text{mod } p)$

$\Rightarrow$ $p \mid a^p - a$

(4) $\frac{(p^2)!}{(p!)^2} = \frac{\frac{p^2(p^2-1)(p^2-2)\ldots(p+1)}{p(p-1)(p-2)\ldots 2.1}}{(p!)(p!)}$

$= \frac{p^2(p^2-1)(p^2-2)\ldots(p+1)}{p!}$

$= \frac{p(p^2-1)(p^2-2)\ldots(p+1)}{(p-1)(p-2)\ldots 2\times 1}$

$\Rightarrow \quad p \Big| \dfrac{(p^2)!}{(p!)^2}$.

option (2) is true.

Q 28. Ans (3) (Topology)

Q 29. Ans (3)

(1) $\dfrac{\mathbb{Z}[i]}{2\mathbb{Z}[i]} \cong \mathbb{Z}_2[i]$

But $\mathbb{Z}_2[i]$ is not field

$\Rightarrow \langle 2 \rangle = 2\mathbb{Z}[i]$ is not maximal ideal in $\mathbb{Z}[i]$

(2) $\dfrac{\mathbb{C}[X,Y]}{X\mathbb{C}[x,y]} \cong \mathbb{C}[Y]$

$\Rightarrow \langle X \rangle = X\,\mathbb{C}[X,Y]$ is not maximal ideal in $\mathbb{C}[X,Y]$

(3) Let

$$I = \begin{Bmatrix} \text{set of polynomials in } \mathbb{C}[x] \\ \text{whose coefficient add up to } 0 \end{Bmatrix}$$

or $I = \{p(x) \in \mathbb{C}[x] \mid p(1) = 0\}$

$\Rightarrow I = \langle X - 1 \rangle$

$\Rightarrow \dfrac{\mathbb{C}[x]}{I} \cong \mathbb{C}$

$\Rightarrow \langle X - 1 \rangle$ is maximal ideal.

(4) $I = \left\langle \sqrt{2} - 1 \right\rangle$

Since $N(x) = N\left(\sqrt{2} - 1\right) = 1$

$\Rightarrow \sqrt{2} - 1$ is invertible element in $\mathbb{Z}\left[\sqrt{2}\right]$

$\Rightarrow \left\langle \sqrt{2} - 1 \right\rangle = \mathbb{Z}\left[\sqrt{2}\right]$

$\Rightarrow I$ is not maximal in $\mathbb{Z}\left[\sqrt{2}\right]$

so, option (3) is true.

Q 30. Ans (2)

$$f(x) = x \log_e\left(1 + \frac{1}{x}\right)$$

$$\Rightarrow \quad f'(x) = \log_e\left(1 + \frac{1}{x}\right) + x \times \frac{1}{\left(1 + \frac{1}{x}\right)}\left(-\frac{1}{x^2}\right)$$

$$= \log_e\left(1 + \frac{1}{x}\right) - \frac{1}{(x+1)}$$

$$\Rightarrow \quad f''(x) = \frac{1}{1 + \frac{1}{x}}\left(-\frac{1}{x^2}\right) + \frac{1}{(x+1)^2} =$$

$$\frac{-1}{x(x+1)} + \frac{1}{(x+1)^2}$$

$$= -\frac{1}{x(x+1)^2} < 0 \quad ; \quad \forall x > 0$$

$\Rightarrow f'(x)$ is strictly decreasing in $(0, \infty)$

$\lim\limits_{x \to 0^+} f'(x) \to \infty$ & $\lim\limits_{x \to \infty} f'(x) \to 0$

$\Rightarrow f'(x) > 0 \;;\; \forall x \in (0, \infty)$

$\Rightarrow f(x)$ increasing in $(0, \infty)$

Also,

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\log\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} \quad \left[\frac{\infty}{\infty} \text{ case}\right]$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{1 + \frac{1}{x}} \times \left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)} = \lim_{x \to 0^+} \frac{x}{x+1} = 0$$

$$\& \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{\log\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} \quad \left[\frac{0}{0} \text{ case}\right]$$

$$= \lim_{x \to \infty} \frac{\frac{1}{1 + \frac{1}{x}}\left(-\frac{1}{x^2}\right)}{-\left(\frac{1}{x^2}\right)} = \lim_{x \to \infty} 1 + \frac{1}{x} = 1$$

So, $f(x)$ is bounded.

Hence option (2) is correct only.

Q 31. Ans (3) (Statistics)

Q 32. Ans (4) (Statistics)

Q 33. Ans (3)

Given A is subring of Q such that for any non-zero rational $r \in Q$, $r \in A$ or $\frac{1}{r} \in A$.

This is the definition of valuation ring of Q

All valuation ring of Q_{arc} of the form

$A_p = \left\{ a/b \in Q : p \nmid b \right\}$ or Q.

So, A is a D.V.R (discrete valuation ring in Q)

(1) $\left\{ a \in A : \frac{1}{a} \notin A \right\} \cup \{0\}$ is an additive subgroup of Q.

In a valuation ring elements with negative valuation form the unique maximal ideal

$m = \{ x \in A : v(x) > 0 \} \cup \{0\}$.

And a maximal ideal of valuation ring is indeed an additive subgroup.

(2) Every valuation ring has exactly one maximal ideal

$\Rightarrow$ A has at most one maximal ideal.

(3) If $A \neq Q$, then A has not infinitely many prime ideals.

A valuation ring of a field is a local P.I.D, hence it has exactly two prime ideals $\{0\}$ and the unique maximal ideal m.

So, it has only one non-zero prime ideal.

(4) For any non-zero $a, b \in A$, a/b or b/a

In a valuation ring, the divisibility relation is total $v(a) \leq v(b) \Rightarrow a/b$

Option (3) is false.

Q 34. Ans (4)

Let $x \& y$ be eigen values of A then

$x + y = tr(A) \Rightarrow x + y = 7$ (1)

Also eigen values of A^2 will be $x^2 \& y^2$

$\Rightarrow$ $x^2 + y^2 = tr(A^2) \Rightarrow x^2 + y^2 = 29$ (2)

$\because$ $\frac{(x+y)^2 - (x^2+y^2)}{2} = xy \Rightarrow xy = \frac{49-29}{2} = 10$

$\Rightarrow$ $\det(A) = 10$

Hence characteristic polynomial of A is

$C(t) = t^2 - (trace\, A)t + \det(A)$

$= t^2 - 7t + 10$

Q 35 Ans. (1)

short cut :

Given $u(x,1) = 1 + x \Rightarrow u(1,1) = 1 + 1 = 2$

So, option (1) is correct.

Aliter:

Auxiliary equation is

$\frac{dx}{y+u} = \frac{dy}{y} = \frac{du}{x-y} = \frac{dx-dy}{u}$ (From (1), (2))

(3) & (4) $\Rightarrow u\, du = (x-y)(dx-dy)$

$\Rightarrow$ $\frac{u^2}{2} = \frac{(x-y)^2}{2} + \frac{c_1}{2}$

$\Rightarrow$ $u^2 - (x-y)^2 = c_1$ (1)

(1) & (3) $\Rightarrow \frac{dx+du}{u+x} = \frac{dy}{y} \Rightarrow$

$\ln(u+x) = \ln(y) + \ln c_2$

$\Rightarrow$ $(u+x)y = c_2 \Rightarrow \frac{u+x}{y} = c_2$ (2)

From (1) & (2)

$u^2 - (x-y)^2 = \phi\left(\frac{u+x}{y}\right)$ (3)

$u(x,1) = 1 + x$ given (3) as

$(1+x)^2 - (x-1)^2 = \phi\left(\frac{1+x+x}{1}\right)$

$\Rightarrow$ $4x = \phi(1+2x) \Rightarrow \phi(t) = 2(t-1)$

$\Rightarrow$ $u^2 - (x-y)^2 = 2\left(\frac{u+x}{y} - 1\right)$ (4)

(1) At $(1,1)$ we get $u^2 = 2u \Rightarrow u = 0$ or 2

(2) At $(2,2)$ we get $u^2 = \frac{2u}{2} \Rightarrow u = 0,1$

(3) At $(3,3)$ we get $u^2 = \frac{2}{3}u \Rightarrow u = 0, \frac{2}{3}$

(4) At $(4,4)$ we get $u^2 = 2\left(\frac{u}{4}\right) \Rightarrow u = 0, \frac{1}{2}$

So, option (1) is correct as possibility of 0 is always ruled out .

Q 36. Ans (1) (Statistics)
Q 37. Ans (2) (Statistics)

Q 38. Ans (2)

$p, q \in I_0^+$ i.e. $p, q \in N$

(A) $\exists$ integer $K \geq 1$ such that $p + K = q$,will be true only if $p < q$.

(B) $\exists$ integer $K \geq 1$ such that $q + K = p$,will be true only if $q < p$.

Hence if $p = q$ then both (A) & (B) will be simultaneously false.

Q 39. Ans (1) (Mechanics)

Q 40. Ans (2)

$F(x, y, y') = y'^2 - 4y^2 + 2xy$

$\Rightarrow \frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$ gives

$-8y - 2x - \frac{d}{dx}(2y') = 0 \Rightarrow 2y'' + 2x + 8y = 0$

$\Rightarrow y'' + 4y = x$ (1)

C.F. is $y = c_1 \cos 2x + c_2 \sin 2x$ &
P.I. is

$$y = \frac{1}{4\left(1 + \frac{D^2}{4}\right)} x = \frac{1}{4}\left(1 - \frac{D^2}{4} + \ldots\right) x = \frac{x}{4}$$

$\Rightarrow y = c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4}$

$y(0) = 0 \Rightarrow c_1 = 0$

$y\left(\frac{\pi}{4}\right) = 1 \Rightarrow c_2 + \frac{\pi}{16} = 1 \Rightarrow c_2 = 1 - \frac{\pi}{16}$

$\Rightarrow y(x) = \left(1 - \frac{\pi}{16}\right)\sin 2x + \frac{x}{4}$

So, option (2) is correct.

Q 41. Ans (2) (Statistics)

Q 42. Ans (3)

If $B = N$ & A is finite set, say $A = \{a_1, a_2, \ldots, a_k\}$ i.e. $|A| = k$ then

$|S_1| = |N| \times |N - 1| \times \ldots |N - (k-1)|$

$= |N|^k \sim |N| = |B|$

So there exist a one-to-one map from B to S_1.

Q 43. Ans (2)

As in Cubic spline $S(x), S'(x), S''(x)$ should be continuous in $[0,4]$, where

$$S(x) = \begin{cases} a(x-2)^2 + b(x-1)^2 ; 0 \leq x \leq 1 \\ (x-2)^2 \quad ; 1 < x \leq 3 \\ 2c(x-2)^2 + (x-3)^3 ; 3 < x \leq 4 \end{cases}$$

$$\Rightarrow S'(x) = \begin{cases} 2a(x-2) + 2b(x-1) ; \ 0 \leq x \leq 1 \\ 2(x-2) \quad ; \ 1 < x \leq 3 \\ 4c(x-2) + 3(x-3)^2 \ ; \ 3 < x \leq 4 \end{cases}$$

$$S''(x) = \begin{cases} 2a + 2b \quad ; \ 0 \leq x \leq 1 \\ 2 \quad ; \ 1 < x \leq 3 \\ 4c + 6(x-3); \ 3 < x \leq 4 \end{cases}$$

For continuity of $S(x)$

$a = 1 \,\&\, 1 = 2c$

For continuity of $S'(x)$

$-2a = -2 \,\&\, 4c = 2$

For continuity of $S''(x); 2a + 2b = 2$ & $4c = 2$

$\Rightarrow b = 0$. So, $2a + b + 2c = 2 \times 1 + 0 + 1 = 3$

Q 44. Ans (2) (Statistics)

Q 45. Ans (4)

$Q(f)=Q(ax^3+bx^2+cx)=Q(a,b,c)$

$=\int_{-1}^{1}(3at^2+2bt+c)^2\ dt$

$=\int_{-1}^{1}(9a^2t^4+4b^2t^2+c^2+12abt^3+$

$6act^2+4bct)dt$

$=2\int_{0}^{1}(9a^2t^4+4b^2t^2+c^2+6act^2)dt$

$=\frac{18}{5}a^2+\frac{8}{3}b^2+2c^2+4ac$

Matrix of Quadratic from Q will be

$$A=\begin{bmatrix}\frac{18}{5} & 0 & 2\\ 0 & \frac{8}{3} & 0\\ 2 & 0 & 2\end{bmatrix}$$

Whose leading principal minors are

$$\left|\frac{18}{5}\right|,\begin{vmatrix}\frac{18}{5} & 0\\ 0 & \frac{8}{3}\end{vmatrix}\&\begin{vmatrix}\frac{18}{5} & 0 & 2\\ 0 & \frac{8}{3} & 0\\ 2 & 0 & 2\end{vmatrix}$$

$=\frac{18}{5},\frac{48}{5}\&\frac{96}{5}-\frac{16}{3}$ i.e. $\frac{18}{5},\frac{48}{5}\&\frac{20}{15}$

Which are all positive, so Q is positive definite Quadratic form on V.

So Range $(Q)=R_0^+=[0,\infty)$, so if takes every positive value.

Also, $Q(x)=Q(0,0,1)=2$

Also, $Q(f+g)=\int_{-1}^{1}(f'(t)+g'(t))^2=dt$

$=\int_{-1}^{1}(f'(t))+(g'(t))^2+(2f'(t)+g'(t))dt$

$=Q(f)+Q(g)+2\int_{-1}^{1}f'(t)g'(t)\ dt$

$\neq Q(f)+Q(g)$.

So , option (4) is false.

Q 46. Ans (4)

$u_{tt}-u_{xx}=0\Rightarrow u_{tt}=u_{xx}$

$\Rightarrow\quad c=1,\ f(x)=1+x^2\ \&\ g(x)=x+1$

For $u(1,1)$; $x=1\ \&\ t=1$

$\Rightarrow\quad x-ct=0\ \&\ x+ct=2$

$\Rightarrow\quad u(1,1)=\frac{f(0)+f(2)}{2}+\frac{1}{2}\int_0^2 g(x)dx$

$=\frac{1+5}{2}+\frac{1}{2}\int_0^2 x+1dx$

$=3+\frac{1}{2}\left[\frac{x^2}{2}+x\right]\Big|_0^2=3+\frac{1}{2}[2+2]=5$

Q 47. Ans (1)

By Gershgorin circle theorem every eigenvalue of complex $n\times n$ matrix A lies within at least one of the gershgorin discs $D(a_{ii},R_i)$ where $R_i=\sum_{j\neq i}|a_{ij}|$

Now as sum of the modulus of elements in each row of A is equal to 101, so modulus of all eigenvalues of A will be atmost 101.

Also sum of elements in each row of A is 101, so 101 is an eigenvalues of A. So largest real eigenvalue of A will be 101.

Q 48. Ans (4)

$\phi(x)=x$ is a solution, so let general solution be $y=vx$ then $\frac{dy}{dx}=v+x\frac{dv}{dx}$ &

$\frac{d^2y}{dx^2}=2\frac{dv}{dx}+x\frac{d^2v}{dx^2}$, putting it into given ODE we get

$$x\frac{d^2v}{dx^2}+2\frac{dv}{dx}-\left(\frac{2}{x^2}+\frac{1}{x}\right)\left(x\left(v+x\frac{dv}{dx}\right)-xv\right)=0$$

$\Rightarrow\quad x\frac{d^2v}{dx^2}+2\frac{dv}{dx}-(2+x)\frac{dv}{dx}=0$

$\Rightarrow\quad x\frac{d^2v}{dx^2}-x\frac{dv}{dx}=0\Rightarrow\frac{d^2v}{dx^2}-\frac{dv}{dx}=0$

$\Rightarrow \quad (D^2 - D)v = 0 \Rightarrow D(D-1)v = 0$

$\Rightarrow \quad v = (c_1 e^{0x} + c_2 e^x) = (c_1 + c_2 e^x)$

So, general solution is $y = vx$

$\Rightarrow \quad y = (c_1 + c_2 x)x$

So, option (4) is correct.

Q 49. Ans (1)

$\phi : X \to R^3$ is given by

$\phi(f) = (f(1), f'(1), f''(1))$ So,

$\ker\phi = \{f \in X \mid f(1) = 0, f'(1) = 0 \,\&\, f''(1) = 0\}$

So $\dim(\ker\phi) = \dim X - 3$ (1)

As $\dim(X)$ is infinite, so dimension of $\ker(\phi)$ is also infinite.

So, options (2) & (4) are false

Also dimension of

$X/\ker\phi = \dim(X) - \dim(\ker\phi)$

$= 3$. So option (1) is correct & option (3) is incorrect.

Q 50. Ans (2)

Given $f(z) = \dfrac{z-i}{z+i} \Rightarrow \dfrac{w}{1} = \dfrac{z-i}{z+i}$

$\Rightarrow \quad \dfrac{w+1}{w-1} = \dfrac{2z}{-2i}$ (By componenedo & dividendo)

$\Rightarrow \quad z = \dfrac{-i(w+1)}{(w-1)} = \dfrac{-i((u+1)+iv)}{(u-1)+iv} = \dfrac{v-i(u+1)}{(u-1)+iv}$

$= \dfrac{[v-i(u+1)][(u-1)-iv]}{(u-1)^2+v^2} = \dfrac{-2v+i(-v^2+1-u^2)}{(u-1)^2+v^2}$

$\Rightarrow \quad x + iy = \dfrac{-2v}{(u-1)^2+v^2} + i\dfrac{(u^2+v^2-1)}{(u-1)^2+v^2}$ (1)

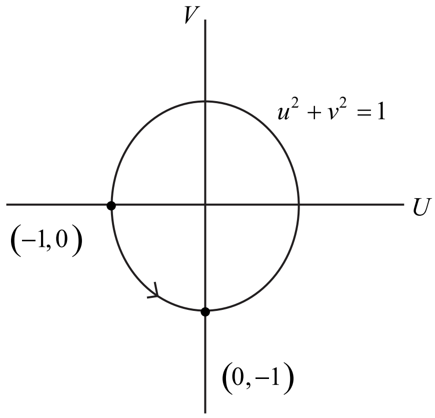
For interval $[0,1]$ we have $x \in [0,1]$ & $y = 0$

$y = 0 \Rightarrow u^2 + v^2 = 1$ from equation (1)

& $x \in [0,1] \Rightarrow 0 \le \dfrac{-2v}{(u-1)^2+v^2} \le 1$

$\Rightarrow \quad 0 \le -2v \le u^2 + v^2 - 2u + 1$

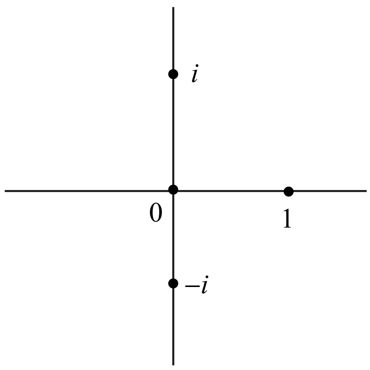
$\Rightarrow \quad v \le 0 \,\&\, (u-1)^2 + (v+1)^2 \ge 1$



So, image of $[0,1]$ i.e.

$X = \left\{1e^{i\theta} \mid \theta \in \left[\pi, \dfrac{3\pi}{2}\right]\right\}$

Shortcut:-



$f(z) = \dfrac{z-i}{z+i} \Rightarrow f(0) = \dfrac{-i}{i} = -1$ &

$f(1) = \dfrac{1-i}{1+i} = \dfrac{-2i}{2} = -i$

As, z lines on perpendicular bisector joining $i \,\&\, -i$, so for them $|w| = f(z) = \left|\dfrac{z-i}{z+1}\right| = 1$ and from -1 to $-i$ amplitude varies from π to $\dfrac{3\pi}{2}$, so $X = \left\{e^{i\theta} \mid \theta \in \left[\pi, \dfrac{3\pi}{2}\right]\right\}$

Q 51. Ans (2) (Statistics)

Q 52. Ans (4)

If $x \in C$ then $x = 0.a_1a_2...a_k$

$$= \frac{a_1a_2...a_k}{10^k} = \frac{a_1a_2...a_k}{2^k.5^k} = \frac{r}{2^m5^n}$$

where $r \in N \,\&\, m,n \in N \cup \{0\}$.

$\Rightarrow x \in B \Rightarrow C \subseteq B$ (1)

Also, $x \in B \Rightarrow x = \frac{p}{2^m\ 5^n} = \frac{p \times 2^{|n-m|}}{10^k}$;

$k = \max(m,n) \Rightarrow x \in C$

$\Rightarrow$ $B \subseteq C$ (2)

(1) & (2) $\Rightarrow B = C$

Also $x \in A \Rightarrow x \in B$, because by putting $n = 0$ in B we get elements of A.

So, $A \subseteq B$, but $0.97 \in B$ but $0.97 \notin A$, so

$A \underset{\neq}{\subset} B = C. \quad \because 0.97 = \frac{97}{2^2.5^2}$

Q 53. Ans (2) (Statistics)

Q 54. Ans (2)

$$f(x) = \frac{3x+2}{4x+3} \ \&$$

$$x_{n+1} = f(x) \Rightarrow x_{n+1} = \frac{3x_n+2}{4x_n+3} \quad (1)$$

Now if $\{x_n\}$ converge to l i.e. $\lim_{n\to\infty} x_n = l$ then $\lim_{n\to\infty} x_{n+1} = l$. Hence (1) becomes

$$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} \frac{3x_n+2}{4x_n+3} \Rightarrow l = \frac{3l+2}{4l+3}$$

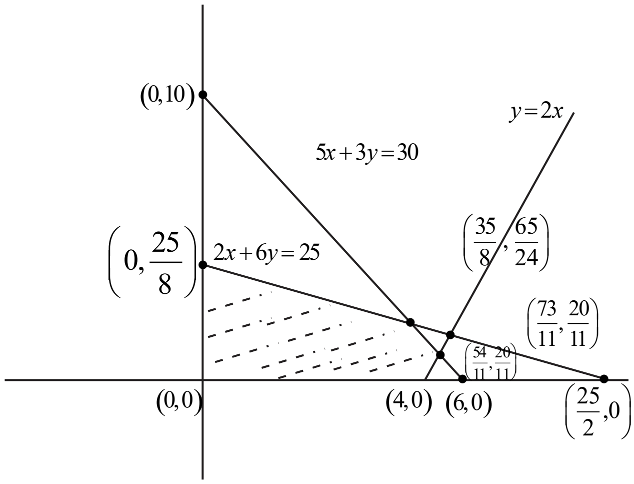
$$\Rightarrow \quad 4l^2 + 3l = 3l + 2 \Rightarrow l^2 = \frac{1}{2} \Rightarrow l = \pm\frac{1}{\sqrt{2}}$$

So if l is positive then $l = \frac{1}{\sqrt{2}}$ &

if l is negative then $l = \frac{-1}{\sqrt{2}}$

So, option (2) is correct.

Q 55. Ans (3)



Vertices of feasible region are

$$(0,0), \left(0,\frac{25}{8}\right), (4,0), \left(\frac{54}{11},\frac{20}{11}\right) \& \left(\frac{35}{8},\frac{65}{24}\right)$$

Objective Function is $z = x + y$

$$z(0,0) = 0, z\left(0,\frac{25}{8}\right) = \frac{25}{8}, z(4,0) = 4,$$

$$z\left(\frac{54}{11},\frac{20}{11}\right) = \frac{74}{11}, z\left(\frac{35}{8},\frac{65}{24}\right) = \frac{170}{24} = \frac{85}{12}$$

So, optimal value of objective function is $\frac{85}{12}$.

So, option (3) is correct.

Q 56. Ans (3) (Statistics)

Q 57. Ans (2)

$f_1(x) = 1$ if $x \in (0,\pi)$

$f_2(x) = 2\cos x$ if $x \in (0,\pi)$

$f_3(x) = 3 - 4\sin^2 x$ if $x \in (0,\pi)$

$$\langle f_1, f_2 \rangle = \int_0^\pi 2\cos x \sin^2 x\, dx = 2\frac{\sin^3 x}{3}\Big|_0^\pi = 0$$

$$\langle f_1, f_3 \rangle = \int_0^\pi (3 - 4\sin^2 x)\sin x\, dx$$

$$= 2\int_0^{\pi/2} (3 - 4\sin^2 x)\sin^2 x\, dx$$

$$= 2\left[\frac{3}{2}\times\frac{\pi}{2} - 4\times\frac{3.1}{4.2}\frac{\pi}{2}\right] = 0$$

$$\langle f_2, f_3\rangle = 2\int\left(3\sin^2 x - 4\sin^4 x\right)\cos x\, dx$$

$$= 2\left[\frac{3\sin^3 x}{3} - \frac{4\sin^5 x}{5}\right]_0^{\pi} = 0$$

$$\langle f_n, f_n\rangle = \frac{2}{\pi}\int_0^{\pi}\sin^2 nx\, dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\frac{1-\cos 2nx}{2}\, dx$$

$$= \frac{2}{\pi}\left[\frac{1}{2}x - \frac{\sin 2nx}{4n}\right]\Bigg|_0^{\pi} = \frac{2}{\pi}\times\frac{\pi}{2} = 1$$

So, each $f_n(x)$ are unit vector

Now, $V = L(\{f_1, f_2, f_3\})$, So $\{f_1, f_2, f_3\}$ is an orthonormal basis of V.

Q 58. Ans (4)

Let probability of occurence of a particular non-prime number be K then probability of occurence of a particular prime number will be $2K$, so

$K + K + K + 2K + 2K + 2K = 1 \Rightarrow K = 1/9$

Set of odd numbers is $\{1,3,5\}$ probability of occurence of odd number will be $\frac{1}{9}+\frac{2}{9}+\frac{2}{9}=\frac{5}{9}$; $3\,\&\,5$ ae prime & 1 is non-prime.

Q 59. Ans (1)

Mclaurin's expansion of entire function will be $f(z) = \sum_{n=0}^{\infty}\frac{f^n(0)}{n!}z^n$; $f^0(0) = f(0)$

As it is entire so it's radius of convergence will be ∞. So, $\lim_{n\to\infty}|a_n|^{1/n} = 0$ i.e.

$$\lim_{n\to\infty}\left|\frac{f^n(0)}{n!}\right|^{1/n} = 0 \qquad (1)$$

For option (1) it is $\lim_{n\to\infty}\frac{1}{n} = 0$. So it is correct.

Q 60. Ans (4)

As each observation is either x or $x+r$, so if these $2n+1$ observations are arrganged in assending order then $(n+1)^{th}$ data is both median and mode.

PART "C"

Q 61. Ans (1,2,3) (Statistics)

Q 62. Ans (1,3,4)

Given D.E. is $\frac{d^2y}{dx^2} + q(x)y = 0$ (1)

(1) $y = \cos x$ is a solution $\Rightarrow y'' = -\cos x$ putting in (1) we get $-\cos x + q(x)\cos x = 0$

$\Rightarrow$ $(q(x)-1)\cos x = 0 \Rightarrow q(x) = 1$

$\Rightarrow$ D.E. (1) is $y'' + y = 0$ whose other L.I. solution is $y = \sin x$

So, option (2) is true & option (1) is false.

(3) Now $e^x \sin x$ is a solution of (1)

$\Rightarrow$ $y = e^x\sin x \Rightarrow y' = e^x(\sin x + \cos x)$

$\Rightarrow$ $y'' = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$

$= 2e^x\cos x$

So, $y'' + q(x) = y = 0 \Rightarrow$

$2e^x\cos x + q(x)e^x\sin x = 0$

$\Rightarrow$ $2\cos x + q(x)\sin x = 0$

$\Rightarrow$ $q(x) = \frac{-2\cos x}{\sin x}$

In this case if $y = e^x\cos 2x$ then

$y' = e^x(\cos 2x - 2\sin 2x)$

$\Rightarrow$ $y'' = e^x(\cos 2x - 2\sin 2x - 2\sin 2x - 4\cos 2x)$

$= e^x(-3\cos 2x - 4\sin 2x)$

Putting in (1) we get

$e^x(-3\cos 2x - 4\sin 2x) - \frac{2\cos x}{\sin x}(e^x\cos 2x)$

$\neq 0$

So, option (3) is false.

(4) $y = xe^x$ is a solution of (1)

$\Rightarrow$ $y' = e^x(x+1) \Rightarrow y'' = e^x(x+2)$

So,

$y'' = q(x)y = 0 \Rightarrow e^x(x+2) + q(x)e^x.x = 0$

$\Rightarrow$ $(x+2) + (q(x))x = 0$

$\Rightarrow$ $q(x) = \dfrac{-(x+2)}{x} = -\left(1+\dfrac{2}{x}\right)$

Now $y = x(x-1)e^x$ will be also solution then

$y' = e^x(x^2 - x + 2x - 1) = e^x(x^2 + x - 1)$

$\Rightarrow$ $y'' = e^x(x^2 + x - 1 + 2x + 1) = e^x(x^2 + 3x)$

So,

$y'' + q(x)y = e^x(x^2 + 3x) - \left(1+\dfrac{2}{x}\right)e^x x(x-1)$

$= e^x(x^2 + 3x - x^2 + x + 1 - x) = e^x(3x+1)$

$\neq 0$

So, option (4) is false.

Q 63. Ans (1,3) (Statistics)

Q 64. Ans (1,2,3) (Statistics)

Q 65. Ans (1,2,4) (Statistics)

Q 66. Ans (2,4) (Statistics)

Q 67. Ans (1,2,4)

$\because$ $f''(x) \geq 0$; $\forall x \in (0,\infty)$

$\therefore$ $f'(x)$ is monotonically increasing in $(0,\infty)$.

Also if at any $C \in (0,\infty), f'(C) > 0$ then $f'(x) > 0$; $\forall x \in (C,\infty)$. So $f(x)$ is unbounded above which is false.

So, $f'(x) \leq 0$, $\forall x > 0$ (Option 1)

As $f(x)$ is bounded in $(0,\infty)$

So, $\lim\limits_{x\to\infty} f(x)$ exist,

so, $\lim\limits_{x\to\infty} f'(x) = 0$ (option 2)

Now $\lim\limits_{x\to\infty} \dfrac{f(x)}{\ln x} = 0 \Rightarrow \lim\limits_{x\to\infty} \dfrac{f'(x)}{(1/x)} = 0$

$\Rightarrow$ $\lim\limits_{x\to\infty} x f'(x) = 0$ (Option 4 is correct & Option 3 is incorrect)

Q 68. Ans (1)

Given $\{f_n(x)\}$ is a sequence of real valued functions over R then

(1) If $\{f_n(x)\}$ is uniformly continuous in R and $\{f_n(x)\}$ converges uniformly to $f(x)$ in R then $f(x)$ is uniformly continuous in R by concept of proof of U.C. & Continuity.

(2) Take $f_n(x) = \dfrac{nx}{n+1} \Rightarrow \lim\limits_{n\to\infty} f_n(x) = f(x) = x$

Hence $\{f_n(x)\}$ is bounded for each $x \in R$ but $f(x) = x$ is unbounded in R.

So, option (2) is incorrect.

(3) Take $f_n(x) = nxe^{-nx^2} = \dfrac{nx}{e^{nx^2}}$ then $\{f_n(x)\}$ is bounded and continuous and converges pointwise to $f(x) = 0$ which is bounded and continuous but convergence in not uniform

$\because$ $f\left(\dfrac{1}{\sqrt{n}}\right) = \dfrac{\sqrt{n}}{e} \leq M_n$

So, M_n does not tends to 0 as $n \to \infty$.

(4) Take $f_n(x) = \sqrt{x^2 + \dfrac{1}{n}} \Rightarrow f(x) = |x|$

Here $f_n(x)$ is differentiable and converges to $f(x) = |x|$ but $f(x) = |x|$ is not differentiable.

Q 69. Ans (3,4)

$f(x) = x^2 + x + 1 \Rightarrow f'(x) = 2x + 1$

$f(x-1) = (x-1)^2 + (x-1) + 1 = x^2 - x + 1$

$f(x+1) = (x+1)^2 + (x+1) + 1 = x^2 + 3x + 3$

$f(x) - f(x-1) = 2x$

$$f(x+1)-f(x)=2x+2$$

(1) $\{f'(x), f(x)-f(x-1), 1\} = \{2x+1, 2x, 1\}$

$\Rightarrow$ 1st vector is sum of 2nd & 3rd vector so it is linearly dependent.

(2) $\{f(x+1)-f(x), f(x)-f(x-1), 1\}$

$=\{2x+2, 2x, 1\}$

$\Rightarrow$ 1st vector is equal to sum of 2nd vector and twice of 3rd vector, so set is linearly dependent.

(3) $\{f(x), f'(x), 1\} = \{x^2+x+1, 2x+1, 1\}$ is linearly independent as no vector can be spanned by remaining two other vectors. Also by matrix representation.

$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 0\\ 1 & 0 & 0\end{bmatrix}$ is non-singular, so it is linearly independent.

(4) $\{f(x+1), f(x-1), f(x)\}$

$=\{x^2+3x+3, x^2-x+1, x^2+x+1\}$ has matrix representation as $\begin{bmatrix}1 & 3 & 3\\ 1 & -1 & 1\\ 1 & 1 & 1\end{bmatrix}$ whose

determinant is $\begin{vmatrix}1 & 3 & 3\\ 1 & -1 & 1\\ 1 & 1 & 1\end{vmatrix} = \begin{vmatrix}1 & 3 & 3\\ 0 & -4 & -2\\ 0 & -2 & -2\end{vmatrix}$

$=4\neq 0$, so it is linearly independent.

Q 70. Ans (1,2)

$$t_n=\sum_{K=0}^{n}\binom{n}{K}\frac{(-1)^K}{n^K}=\sum_{K=0}^{n}\binom{n}{K}\left(-\frac{1}{n}\right)^K$$

$$=\left(1-\frac{1}{n}\right)^n \ \& \lim_{n\to\infty} t_n=\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n \quad \left[1^\infty \text{case}\right]$$

$$= e^{\lim_{n\to\infty} n\left(1-\frac{1}{n}-1\right)}=e^{-1}=\frac{1}{e}$$

So, option (4) is false.

Also, $S_n=\sum_{K=0}^{n}\frac{1}{(K!)^2}=1+1+\frac{1}{(2!)^2}+\frac{1}{(3!)^2}+\ldots$

$\Rightarrow$ $\lim S_n>2$ (i)

Also $\frac{1}{(K!)^2}\leq\frac{1}{K!}$

$\Rightarrow$ $\sum_{K=0}^{n}\frac{1}{(K!)^2}\leq\sum_{K=0}^{n}\frac{1}{K!}$

$\Rightarrow$ $\lim_{n\to\infty} S_n=\lim_{n\to\infty}\sum_{K=0}^{n}\frac{1}{(K!)^2}\leq\lim_{n\to\infty}\sum_{K=0}^{n}\frac{1}{K!}=e$

$\Rightarrow$ $\lim_{n\to\infty} S_n\leq e$ (ii)

$\Rightarrow$ $2\leq\lim_{n\to\infty} S_n\leq e$

So, option (3) is incorrect but options (1) & (2) are correct.

Q 71. Ans (3,4)

If $f(x)=[x]$ and

$$g(x)=0 \ ; x\in Q^c\cup\{0\}$$
$$=\frac{1}{q}; x=\frac{p}{q}; G.C.D(p,q)=1$$

then $f\circ g=1 \ ; x\in Q\setminus\{0\}$
$=0 \ ; x\in Q^c\cup\{0\}$

So, $f \,\&\, g\in R[0,1]$ but $f\circ g\notin R[0,1]$

So, option (1) is incorrect.

Take $f(x)=1$ &

$$g(x)=1; \ x=0$$
$$=x; \ \text{otherwise}$$

then

$$\frac{f(x)}{g(x)}=1 \quad ; \ x=0$$
$$=\frac{1}{x} \ ; \text{otherwise}$$

Here $f(x)\,\&\, g(x)\in R[0,1]$ but

$\frac{f(x)}{g(x)} \notin R[0,1]$.

Also, if $f \,\&\, g$ are Riemann integrable then $f^2 \,\&\, g^2$ is also Riemann integrable so f^2+g^2 is also Riemann integrable.

Also as $f^2+g^2 \geq 0$ so, $\sqrt{f^2+g^2}$ is also Riemann integrable on $[a,b]$.

Further if $f(x)$ and $g(x)$ are continuous and hence it is then $f \circ g$ is also continuous Riemaan integrable.
So, option (3) & (4) are correct.

Q 72. Ans (2,3)

(1) If we take $f(x)=1 \,\&\, g(x)=0$ then

$T(f(x)) = \int_0^x 1\,dx = x$ &

$T(g)(x) = \int_0^x 0\,dx = 0$

$\Rightarrow$ $\|T(f)-T(g)\| = \|x-0\| = \|x\| = \sup_{x\in[0,1]} x = 1$

$\|f-g\| = \|1-0\| = \|1\| = 1$, so $\nexists\, \alpha \in (0,1)$ such that $\|T(f)(x)-T(g)(x)\| \leq \alpha \|f-g\|$

$\because$ $1 \leq \alpha.1$ is not true for any $\alpha \in (0,1)$.
So, option (1) is false.

(2) Option (2) is true as $\exists \alpha \in (0,1)$ such that $\|T^2(f)-T^2(g)\| \leq \alpha \|f-g\|$ because for above example of $f(x)$ & $g(x)$, $T^2(f) = \frac{x^2}{2}$ &

$T^2(g) = 0 \Rightarrow \|T^2(f)-T^2(g)\| = \sup_{x\in[0,1]} \frac{x^2}{2} = \frac{1}{2}$

& $\frac{1}{2} \leq \alpha.1$ for $\alpha \in (0,1)$.

(3) $T(f) = f \Rightarrow \int_0^x f(t)\,dt = f(x)$. Now differentiating both sides we get,

$f(x) = f'(x) \Rightarrow (D-1)f(x) = 0$

$\Rightarrow$ $f(x) = ce^x \Rightarrow T(f(x)) = \int_0^x c\,e^t\,dt = c(e^x-1)$. So $T(f(x)) = f(x)$

$\Rightarrow$ $c(e^x-1) = c\,e^x \Rightarrow -c = 0 \Rightarrow c = 0 \Rightarrow$ The set $\{f \in c[0,1] : T(f) = f\}$ is singleton set.
So option (3) is correct.

(4) $f(x) = 1 \Rightarrow T^n f(x) = \frac{x^n}{n!}$ &

$\|T^n\| = \sup_{x\in[0,1)} \frac{x^n}{n!} = \frac{1}{n!}$ as $n \to \infty$, $\|T\|^n \to 0$

So, option (4) is false.

Q 73. Ans (1,4)

$u(x,y) = 1 + 2x^2y^2 \geq 1 + 2(0) = 1$

Also on the boundary $x^2+y^2=1$

$x = \cos\theta, y = \sin\theta \Rightarrow$

$u(x,y) = u(\cos\theta, \sin\theta) =$

$1 + 2\cos^2\theta \sin^2\theta = 1 + \frac{(\sin 2\theta)^2}{2}$

$\leq 1 + \frac{1^2}{2} = \frac{3}{2}$

$\Rightarrow$ $1 \leq u(x,y) \leq \frac{3}{2}$

So, minimum value of $u(x,y)$ is 1 & maximum value of $u(x,y)$ is $3/2$.
So, options (1) & (4) are correct.

Q 74. Ans (1) (Statistics)

Q 75. Ans (1,2,3,4) (Markov)

Q 76. Ans (4) (Statistics)

Q 77. Ans (1,2,3)

Given $f : \mathbb{Q}[x] \to \mathbb{Q}[x]$ ring homomorphism and $f(1) = 1$.

(1) f is onto $\Rightarrow f^n$ is onto; $\forall n \geq 1$
(Composition of onto maps is onto)

(2) The chain of ideal $\ker f \subseteq \ker f^2 \subseteq \ker f^3 \subseteq \ldots$ is ascending. Since $\mathbb{Q}[x]$ is Noetherian ring, so any ascending chain of ideals must stabilize.
$\exists n \in \mathbb{N}$ such that $\ker f^n = \ker f^{n+1}$

(3) In a Noetherian ring a surjective homomorphism onto itself is an isomorphism.

$\Rightarrow$ f is onto $\Rightarrow f$ is one-one

(4) Let $f : \mathbb{Q}[x] \to \mathbb{Q}[x]$ defined $x \to x^2$.
Then $\text{Range}(f) = \mathbb{Q}[x^2] \neq \mathbb{Q}[x]$

$\Rightarrow$ f is one-one but not onto
Option (1,2,3) are correct.

Q 78 Ans (1,4)

(1) Is true as preimage of compact set under non-constant polynomial is compact.

(2) Is false as for $f(x) = x^2$, $\{1\}$ is connected but it's preimage $\{1, -1\}$ is not connected.

(3) Is false because for $x = 0$ from R has $\in$-nbd as $N_{\in}(0) = \{x \in R \mid -\in < x < \in\}$ is not homeomorphic to open set $(0,1) \cup (2,3)$ from R.

(4) Is correct because nonconstant polynomial maps bounded set onto bounded set.

$\because$ If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ and if $|x| \leq M$ then

$|f(x)| \leq |a_n||x|^n + |a_{n-1}||x|^{n-1} + \ldots + |a_1||x| + |a_0|$

$= |a_n| M^n + |a_{n-1}| M^{n-1} + \ldots + |a_1| M + |a_0|$

Q 79. Ans (1,2)

(1) $G_1 \trianglelefteq G$ and $G_2 \trianglelefteq G$
By second isomorphism Theorem, we have

$$\frac{G_2 G_1}{G_1} \cong \frac{G_2}{(G_2 \cap G_1)}$$

(2) Let $H_i \trianglelefteq G_i$. Then

$$\frac{G_1 \times G_2 \times \ldots \times G_n}{H_1 \times H_2 \times \ldots \times H_n} \cong \frac{G_1}{H_1} \times \frac{G_2}{H_2} \times \ldots \times \frac{G_n}{H_n}$$

(Result)

(3) If $G_1 \trianglelefteq G_2$ and $G_2 \trianglelefteq G$ then it doesn't imply $G_1 \trianglelefteq G$.

e.g.:- $\{R_0, H\} \trianglelefteq \{R_0, H, V, R_{180}\} \trianglelefteq D_4$.
But $\{R_0, H\} \ntrianglelefteq D_4$

(4) Subgroup of prime index needn't be normal

e.g.:- Let $H = \langle (12) \rangle = \{I, (12)\} \leq S_3$, and index of H in S_3 is 3.
But $H \ntrianglelefteq S_3$ (H is not normal in S_3)
So, option (1), (2) are correct.

Q 80. Ans (2,3)

$f(tx_1, tx_2) = t^3 f(x_1, x_2) \Rightarrow f(x_1, x_2)$ is homogeneous function in x_1 & x_2 of degree $3(=n)$. And by Cauchy Euler Theorem on homogeous function.

$x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} = n f ; \forall (x, y)$, so at $(1, -1)$ it is $\frac{\partial f}{\partial x_1}(1, -1) - \frac{\partial f}{\partial x_2}(1, -1) = 3 f(1, -1)$

So, option (2) is correct.
Also,

$$x_1^2 \frac{\partial^2 f}{\partial x_1^2}(x_1, x_2) + x_2^2 \frac{\partial^2 f}{\partial x_2^2}(x_1, x_2) + 2x_1 x_2 \frac{\partial^2 f}{\partial x_1 \partial x_2} =$$

$$n(n-1) f = 3(3-1) f = 6 f(x_1, x_2)$$

So, option (4) is correct.
Also option (1) & (3) are incorrect.

Q 81. Ans (4)

(1) $Aut(G_1) \cong Aut(G_2) \not\Rightarrow G_1 \cong G_2$

e.g.:- $Aut(\mathbb{Z}_{10}) \cong Aut(\mathbb{Z}_5) \cong \mathbb{Z}_4$
But $\mathbb{Z}_{10} \ncong \mathbb{Z}_5$

(2) If $|G| = 2 \Rightarrow G \cong \mathbb{Z}_2$, and $Aut(\mathbb{Z}_2 \times \mathbb{Z}_2) \cong S_3$

But S_3 is non-abelian.

(3) Let $G=(\mathbb{C},+)$. Then

$Aut(\mathbb{C})$ is non-abelian group.

Let $f:\mathbb{C}\to\mathbb{C}$ given by $f(z)=\overline{z}$ and $g:\mathbb{C}\to\mathbb{C}$ given by $g(z)=a\overline{z}$.

Since $f,g\in Aut(\mathbb{C})$ and

$f\circ g=f(g(z))=f(a\overline{z})=\overline{a}z$ and

$g\circ f=g(f(z))=g(\overline{z})=a\,z$

But in general $f\circ g\neq g\circ f$.

(4) If G is finite $\Rightarrow Aut(G)$ is finite. But not conversly

Q 82. Ans (2)

$f(x)=x^5+x+1\in\mathbb{Q}[x]$

$g(x)=x^5-x+1\in\mathbb{Q}[x]$

Since

$f(x)=x^5+x+1=(x^2+x+1)(x^3-x^2+1)$

in $\mathbb{Q}[x]$

But neither (x^2+x+1) nor (x^3-x^2+1) is unit in $\mathbb{Q}[x]$

$\Rightarrow$ $f(x)$ is not irreducible in $\mathbb{Q}$

Next, $g(x)=x^5-x+1\in\mathbb{Q}[x]$

By mod 3 Test:-

Since $g(0)=1\neq 0$

$g(1)=1\neq 0$

$g(2)=31\neq 0$ in $\mathbb{Z}_3$

$\Rightarrow$ $g(x)$ has no linear factor in $\mathbb{Z}_3[x]$

$\Rightarrow$ Possible factor of $g(x)$ is of degree 2 and 3.

Possible degree 2 irreducible factors are x^2+1, x^2+x+2 and x^2+2x+2.

By using the factor theorem, we see that x^2+1, x^2+x+2 and x^2+2x+2 are not factor of $g(x)$.

By mod-3 Test:-

$g(x)$ is irreducible over $\mathbb{Z}_3$

$\Rightarrow$ $g(x)$ is irreducible over $\mathbb{Q}$

So, option (2) is correct.

Q 83. Ans (2,3)

(1) Is false as $f(z)=\sin 2\pi z$ is non-constant entire function satisfying $f(z)=f(z+1)$

(2) Is true as $f(z)=f(z+1)=f(z+i)$; $\forall\, z\in c$ implies that $f(z)$ has periods $1\,\&\,i$.

As $\{1,i\}$ is a basis of $C(R)$, so $f(z)$ is constant function.

(3) Is true because $f\left(\frac{1}{z}\right)$ has removable singularity at 0 only if it's Taylor's expansion about 0 has no positive power of z, so $f(z)$ is constant function.

(4) Is false as $f(z)=e^z$ is non-constant entire function but $f\left(\frac{1}{z}\right)$ do not have pole at $z=0$ infact it has essential singularity at $z=0$.

Q 84. Ans (1,2,3)

Given Volterra Integral equation is

$u(x)=x^2+4\int_0^x (t-x)^2 u(t)\,dt$

Here $f(x)=x^2$; $\lambda=4$ and

kernel $K(x,t)=(t-x)^2$

$\because$ $u(x)=L^{-1}\left[\dfrac{L(f(x))}{1-\lambda L(f(x))}\right]$

$$=L^{-1}\left[\frac{L(x^2)}{1-4L(x)^2}\right]=L^{-1}\left[\frac{\frac{2}{s^3}}{1-\frac{8}{s^3}}\right]$$

$$= L^{-1}\left[\frac{2}{s^3-8}\right] = L^{-1}\left[\frac{2}{(s-2)(s^2+2s+4)}\right]$$

$$= L^{-1}\left[\frac{1/6}{s-2} + \frac{-\frac{1}{6}s - \frac{2}{3}}{s^2+2s+4}\right]$$

$$= L^{-1}\left[\frac{1}{6}\left(\frac{1}{s-2}\right) - \frac{1}{6}\left(\frac{(s+1)+3}{(s+1)^2+3}\right)\right]$$

$$= \frac{1}{6}e^{2x} - \frac{e^{-x}}{6}L^{-1}\left(\frac{s+3}{s^2+3}\right)$$

$$\Rightarrow \quad u(x) = \frac{1}{6}e^{2x} - \frac{e^{-x}}{6}\left[\cos\sqrt{3}x + \sqrt{3}\sin\sqrt{3}x\right]$$

(1) $u(0) = \frac{1}{6} - \frac{1}{6} = 0$.

So, option (1) is true.

(2) $u\left(\frac{2\pi}{\sqrt{3}}\right) = \frac{1}{6}e^{\frac{4\pi}{\sqrt{3}}} - e^{\frac{-2\pi}{\sqrt{3}}}(1+0)$

$$= \frac{1}{6}\left(e^{\frac{4\pi}{\sqrt{3}}} - e^{-\frac{2\pi}{\sqrt{3}}}\right)$$

So, option (2) is correct.

(3) $u\left(\frac{\pi}{2\sqrt{3}}\right) = \frac{1}{6}e^{\frac{\pi}{\sqrt{3}}} - \frac{e^{-\frac{\pi}{2\sqrt{3}}}}{6}\left(0+\sqrt{3}\right)$

$$= \frac{1}{6}\left(e^{\frac{\pi}{\sqrt{3}}} - \sqrt{3}\,e^{\frac{-\pi}{2\sqrt{3}}}\right)$$

So, option (3) is correct and option (4) is false.

Q 85. Ans (1,2) (Statistics)

Q 86. Ans (1,3)

Let

$$X = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} \Rightarrow X X^T = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}\begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix}$$

$$\Rightarrow \quad XX^T = \begin{pmatrix} x_1^2+x_2^2 & x_1x_3+x_2x_4 \\ x_1x_3+x_2x_4 & x_3^2+x_4^2 \end{pmatrix};$$

Let $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$

$$DF_A(X) = AX^T + XA^T$$

$$= \begin{pmatrix} 2x_1a_1 & a_1x_3 \\ a_1x_3 & 0 \end{pmatrix} + \begin{pmatrix} 2x_2a_2 & a_2x_4 \\ a_2x_4 & 0 \end{pmatrix} + \begin{pmatrix} 0 & x_1a_3 \\ x_1a_3 & 2x_3a_3 \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & x_2a_4 \\ x_2a_4 & 2x_4a_4 \end{pmatrix} =$$

$$\begin{pmatrix} 2x_1a_1+2x_2a_2 & a_1x_3+a_2x_4+a_3x_1+a_4x_2 \\ a_1x_3+a_2x_4+a_3x_1+a_4x_2 & 2x_3a_3+2x_4a_4 \end{pmatrix}$$

If we take $A = 0$ then $DF_A(X) = 0$

$\Rightarrow$ $DF_A(X)$ is of Rank 0, and $\dim(s) = \frac{2\times 3}{2} = 3$

so it is not surjective, so option (4) is false.

From the above $DF_A : M_2(R) \to S$ is surjective iff A is invertible.

So, option (1) & (3) are correct & (2) is false.

Now matrix of the transformation is

$$\begin{bmatrix} 2a_1 & a_3 & a_3 & 0 \\ 2a_2 & a_4 & a_4 & 0 \\ 0 & a_1 & a_1 & 2a_3 \\ 0 & a_2 & a_2 & 2a_4 \end{bmatrix} \text{By } C_1 \to \frac{1}{2}C_1, \quad C_4 \to \frac{1}{2}C_4 \;\&\; C_3 \to C_3 - C_2$$

$$\sim \begin{bmatrix} a_1 & a_3 & 0 & 0 \\ a_2 & a_4 & 0 & 0 \\ 0 & a_1 & 0 & a_3 \\ 0 & a_2 & 0 & a_4 \end{bmatrix}$$ Whose rank is 3 iff

$\begin{vmatrix} a_1 & a_3 \\ a_2 & a_4 \end{vmatrix} \neq 0$, i.e. A is non-singular.

Q 87. Ans (1,3,4) (Statistics)

Q 88. Ans (1,2,4)

$\because$ $\text{trace}(ST) = \text{trace}(TS)$

$\therefore$ $\text{trace}(ST - TS) = tr(ST) - tr(TS) = 0$

So, if $\lambda_1 \& \lambda_2$ are eigenvalues of $ST-TS$ then $\lambda_1+\lambda_2=0 \Rightarrow \lambda_2=-\lambda_1$, so eigenvalues are $ST-TS$ are $\lambda_1 \& -\lambda_1$, so characteristic polynomial of $ST-TS$ will be $(x-\lambda_1)(x+\lambda_1)=x^2-\lambda_1^2=C(x)$, so by Cayley Hamilton theorem. $C(ST-TS)=0 \Rightarrow (ST-TS)^2-\lambda_1^2 I=0$

$\Rightarrow$ $(ST-TS)^2=\lambda_1^2 I=\lambda I$ (where $\lambda_1^2=\lambda$)

(Option 1)

Also if $\lambda_1 \& -\lambda_1$ are eigenvalues of $ST-TS$ then λ_1^2, λ_1^2 are eigenvalues of $(ST-TS)^2$ by taking $\lambda_1^2=\lambda$ we will get characteristic polynomial of $(ST-TS)^2$ as $(x-\lambda)^2$

(Option 2)

If $ST-TS$ has only one eigenvalue then $\lambda_1=-\lambda_1 \Rightarrow \lambda_1=0$, so both eigen values of $ST-TS$ will be 0 hence it's characteristic polynomial will be $C(x)=(x-0)^2=x^2$, so by Cayley Hermilton Theorem $(ST-TS)^2=0$

So, option (4) is correct.
But option (3) is false.

Q 89. Ans (1,2)

Option (1) is correct as A is invertible matrix then $A^{-1}(AB)A=BA$, so $AB \& BA$ are similar matrices , so they have same minimal polynomial.

Option (2) is correct as eigenvalues of $AB \& BA$ are same. So if 0 is an eigen value of AB then 0 is also an eigen value of BA.

Option (3) is false because if we take $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} \& B=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$ then

$AB=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ here 0 is only eigen value of A & B but eigen values of AB are 1 & 0.

Option (4) is false as if we take $A=0$ & $B=diag[1\ \ 0]$ then $AB=BA=0$, so AB & BA have same minimal polynomial but neither A nor B is invertible.

Q 90. Ans (1,3,4)

Given that f is entire function and it is not polynomial.

So, $A=\{\alpha \in C \mid f^n(\alpha) \neq 0, \forall n \geq 0\}$ is uncountable set because $f(z)$ is transcendental function which is combination of e^z, $\sin z$ $\cos z$ etc. Which have all order derivatives non-zero for uncountable $\alpha \in C$.

So, option (1,3 & 4) are correct.

Q 91. Ans (1,2,3,4)

Vector space is $V_3(Z_3)$ which is of dimension 3 & W is 2 dimensional subspace of $V_3(Z_3)$.

We know that number of r dimensional subspaces of

$$V_n(Z_p)=\frac{(p^n-1)(p^n-p)\dots(p^n-p^{r-1})}{(p^r-1)(p^r-p)\dots(p^r-p^{r-1})}$$

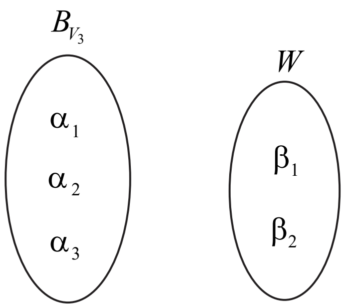
So, (1) Number of 2 dimensional subspaces of V is $\frac{(3^3-1)(3^3-3)}{(3^2-1)(3^2-3)}=\frac{26\times 24}{8\times 6}=13$

(3) Number of 1 dimensional subspace of V is $\frac{(3^3-1)}{(3-1)}=13$

Number of vectors in W is equal to 3^2.

(4) We know that number of linear transformation from $V_n(Z_p)$ from $V_m(Z_p)$ is equal to $(p^m)(p^m)\dots(p^m)$, n times $=(p^m)^n=p^{mn}$, because every element of basis of $V_n(Z_p)$ has p^m options (Cardinality of codomain). Hence number of linear transformations from V to $W=(3^2)^3=3^6=729$

(2)



$\dim V = 3 \;\&\; \dim W = 2$

So, in surjective linear transformation from V to W by Rank-Nullity theorem.

Rank (T) + Nullity (T) = dim V ; where

Rank (T) = dim W = 2 $\Rightarrow$ Nullity (T) = 1

So, in surjective linear transformation from V to W, 1 dimensional subspace will be transformed to zero vector & then remaining 2 dimensional subspace will be bijective with W. So, number of surjective linear transformations from V to W = (No. of 1 dimensional subspace of V) $\times$ (No. of bijections from a 2D subspace of V to W)

$$=\frac{(3^3-1)}{(3-1)}\times(3^2-1)(3^2-3)$$

$$=13\times8\times6=624.$$

So, all 4 options are correct.

Q 92. Ans (1)

$$\int_0^3 f(x)\,dx=\frac{3}{2}\left[f(\alpha)+f(\alpha+\beta)\right] \qquad (1)$$

holds true for polynomials of degree less than or equal to 2. So as basis of $P_2(R)$ is $\{1,x,x^2\}$

(1) Take $f(x)=1$

So, (1) becomes $\int_0^3 1\,dx=\frac{3}{2}[1+1]\Rightarrow 3=3$

(2) Take $f(x)=x$

So, (1) becomes

$$\int_0^3 x\,dx=\frac{3}{2}[\alpha+\alpha+\beta]$$

$$\Rightarrow \quad \frac{9}{2}=\frac{3}{2}[2\alpha+\beta]\Rightarrow 2\alpha+\beta=3 \qquad (2)$$

(3) Now take $f(x)=x^2$,

So, (1) becomes

$$\int_0^3 x^2\,dx=\frac{3}{2}\left[\alpha^2+(\alpha+\beta)^2\right]$$

$$\Rightarrow \quad 9=\frac{3}{2}\left[2\alpha^2+\beta^2+2\alpha\beta\right]$$

$$\Rightarrow \quad 2\alpha^2+\beta^2+2\alpha\beta=6 \qquad (3)$$

From (2), $\beta=3-2\alpha$, Putting in (3) we get

$$2\alpha^2+(3-2\alpha)^2+2\alpha(3-2\alpha)=6$$

$$\Rightarrow \quad 2\alpha^2-6\alpha+3=0$$

$$\Rightarrow \quad \alpha=\frac{6\pm\sqrt{36-24}}{4}=\frac{6\pm2\sqrt{3}}{4}=\frac{3\pm\sqrt{3}}{2}$$

So,

(i) $\alpha=\frac{3-\sqrt{3}}{2}\Rightarrow\beta=3-2\alpha=3-\left(3-\sqrt{3}\right)=\sqrt{3}$

(ii) $\alpha=\frac{3+\sqrt{3}}{2}\Rightarrow\beta=3-2\alpha=3-\left(3+\sqrt{3}\right)=-\sqrt{3}$

So, option (1) is correct.

Q 93. Ans (3,4) (Statistics)

Q 94. Ans (1,3) (Statistics)

Q 95. Ans (1,2)

Given G: group and $H\le G$

$T=\{gH/g\in G\}$ and S_T : Set of all permutations of T, $\pi:G\to S_T$ defined by $\pi(g)(g_1H)=g\,g_1H$

Result:- Note that π is a group homomorphism and $\ker\pi=\bigcap_{x\in G} xHx^{-1}\ \ \forall x\in G$.

Also, $\ker\pi$ is largest normal subgroup contained in H.

(i) $G=GL_2(\mathbb{F}_2)$ and $H\le G$ with $O(H)=p$. Since

$$O(G)=(p^2-1)(p^2-p)=p(p-1)(p^2-1)$$

$=p(p-1)^2(p+1)$ and $O(H)/O(G)$ but $p^2\nmid O(G)$

$\Rightarrow$ H is p-S.S.G i.e.of order p

$\Rightarrow$ Number of p-S.S.G i.e. $\eta_p = 1 + pk$ with $\eta_p / O(G)$

$\Rightarrow$ $\eta_p > 1$ (Think)

$\Rightarrow$ H is not normal subgroup of G

$\Rightarrow$ $\ker\pi = \bigcap_{x\in G} x H x^{-1} =$ largest normal subgroup contained in H.

$\because$ $O(H) = p$

$\Rightarrow$ $O(\ker\pi) = 6$

$\Rightarrow$ $\ker\pi = \{e\}$ (Trivial)

Similarly, option (2) is also correct.

option (4) choose prime $p = 5$ we have

$$O(G) = \frac{O(GL_2(\mathbb{F}_5))}{O(SL_2(\mathbb{F}_5))} = 4$$

$\Rightarrow$ $G \cong \mathbb{Z}_4$ or $G \cong K_4$

Let H be any subgroup of order 2 in G. Then

$$\ker\pi = \bigcap_{x\in G} x Hx^{-1} = \bigcap_{x\in G} H x x^{-1}$$

$$= \bigcap_{x\in G} H e = H$$

[$\because$ Every subgroup of group of order 4 are normal]

$\Rightarrow$ $\ker\pi = H \neq \{e\}$

Similarly, we can always choose same prime p for option (3), for which $\ker\pi \neq \{e\}$

So, option (1) & (2) are true.

Q 96. Ans (2,3,4)

$y' + y = 0 \Rightarrow y' = -y$; $y(0) = 1 = y_0$

$\Rightarrow$ $f(x,y) = -y$. So, Euler's forward method is

$y_n = y_{n-1} + h f(x_{n-1}, y_{n-1}); 0 < h < 1$

$\Rightarrow$ $y_n = y_{n-1} + h(-y_{n-1}) = (1-h) y_{n-1}; n = 1,2,3,...$

$\Rightarrow$ $y_n = (1-h)^2 y_{n-2} = (1-h)^3 y_{n-3} ... = (1-h)^n y_0$

$\Rightarrow$ $y_n = (1-h)^n; n = 0,1,2,...$; $0 < h < 1$

$\Rightarrow$ $|1-h| < 1$ & $0 \le y_n = (1-h)^n \le 1$

So, sequence, $\{y_n\} = \{(1-h)^n\}$ converges to 0.

$\because$ $\lim_{n\to\infty} y_n = \lim_{n\to\infty} (1-h)^n = 0$

& $|y(nh) - y_n| \to 0$ as $n \to \infty$

So, option (2), (3) & (4) are correct.

Q 97. Ans (2,3,4) (Statistics)

Q 98. Ans (1,3)

$(T-2I)^2 (T-3I)^2 (T-5I)^2 = 0 \Rightarrow$ an annihilating polynomial of T is

$f(x) = (x-2)^2 (x-3)^2 (x-5)^2$.

As minimal polynomial $m(x)$ of T is divisor of every annihilating polynomial of T, hence

$m(x) | (x-2)^2 (x-3)^2 (x-5)^2$.

Also, order of T is 7 and 2,3 & 5 are its eigenvalues, so multiplicity of at least one eigenvalue will be at least 3 and it's power in $m(x)$ will be at most 2 so this eigenvalue will have at least 2 Jordan blocks and remaining eigenvalues will have at least 1 Jordan blocks, so in Jordan canonical form T has at least 4 Jordan block and we know that number of L.I. eigen vectors of T is equal to number of Jordan blocks of T. So, T has at least four L.I. eigen vectors. So, option (1) is correct.

If $T = diag[2\ 2\ 2\ 3\ 3\ 3\ 5]$ then

$T - 5I = diag[-3\ -3\ -3\ -2\ -2\ -2\ 0]$

$\Rightarrow$ $(T-5I)^K = diag[(-3)^K (-3)^K (-3)^K (-2)^K (-2)^K (-2)^K\ 0]$

then $W = \{V \in C^7 : (T-5I)^K V = 0; K \in N\}$

$= \{(0,0,0,0,0,0,a)^t \mid a \in C\}$

Whose dimension is 1, so $\dim W \ge 2$ is false, So, option (2) is false.

Now as order of $(T-2I) = 7$, so

$(T-2I)^K, \forall K \ge 7$ has it's rank equal to

Rank $(T-2I)^7$

So, Rank $(T-2I)^{2025}$ = Rank $(T-2I)^{2026}$

$\Rightarrow$ Nullity $(T-2I)^{2025}$ = Nullity $(T-2I)^{2026}$

$\Rightarrow$ Ker $\left((T-2I)^{2025}\right)$ = ker$\left((T-2I)^{2026}\right)$

So, option (3) is correct.

Also, If $T = diag[2\ 2\ 3\ 3\ 5\ 5\ 5]$ then

$(T-2I)(T-3I) = diag[0\ 0\ 0\ 0\ 6\ 6\ 6]$

Also,

$((T-2I)(T-3I))^n = diag\left[0\ 0\ 0\ 0\ 6^n\ 6^n\ 6^n\right]$

$\neq 0$; $\forall n \in N$, so $(T-2I)(T-3I)$ is not nilpotent operator.

So, option (4) is false.

Q 99. Ans (1,3)

Given

$$A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

is in Jordan canonical form whose characteric polynomial & minimal polynomial $C_A(x)$ & $m_A(x)$ are

$$C_A(x) = m_A(x) = (x-1)^2(x-1)^2 = (x^2-1)^2$$

(1)

$$B = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$ is companion matrix

whose characteristic polynomial and minimal polynomial is

$C_B(x) = m_B(x) = x^4 - 2x^2 + 1 = (x^2-1)^2$

So, option (1) is correct.

(2) $$C = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$ is companion matrix

whose characteristic and minimal polynomial are $C_C(x) = m_C(x) = x^4 + 2x^2 - 1$

So, option (2) is false.

(3)

$$D = \left(\begin{array}{cc|cc} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)$$ is upper triangular

block matrix whose eigen values are eigen values of $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ & $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ i.e. $1, -1 \,\&\, 1, -1$

Also G.M. of e.v. 1 $= 4 -$ Rank $(D - 1I)$

$= 4 - 3 = 1$

$\because$

$$(D-1I) = \begin{pmatrix} -1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

By $R_2 \leftarrow R_2 + R_1$ & $R_4 \leftarrow R_4 + R_3$ $\sim \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

By $R_3 \leftarrow R_3 + R_2$

So, Rank $(D) = 3$

G.M. of eigenvalue $-1 = 4 -$ Rank $(D - 1I)$

$= 4 - 1(A + I) = 4 - 3 = 1$

$\because$

$$D + I = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

By $R_2 \leftarrow R_2 - R_1$

& $R_4 \leftarrow R_4 - R_3$

$$\sim\begin{pmatrix}1&1&1&0\\0&0&-1&1\\0&0&0&2\\0&0&0&0\end{pmatrix} \text{ By } R_3 \leftarrow R_3 + R_2$$

So, Rank $(D+I)=3$

So both eigen value 1 & -1 has same G.M. 1, So they have only one Jordan block, So C.P. & m.p. of D are same as

$$C_D(x)=m_D(x)=(x-1)^2(x+1)^2=(x^2-1)^2$$

So, option (3) is true.
For option (4),

$$E=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} \Rightarrow E^2=I$$

and eigenvalues of E are eigenvalues of $\begin{pmatrix}0&1\\1&0\end{pmatrix}$ & $\begin{pmatrix}0&1\\1&0\end{pmatrix}$ i.e. $1,-1,1\,\&\,-1$.

So, characteristic polynomial of E is

$$C_E(x)=(x-1)^2(x-1)^2=(x^2-1)^2$$

and minimal polynomial of E is

$$m_E(x)=(x^2-1)$$

So, option (4) is false.

Q 100. Ans (2)

$$f_n(x)=\frac{e^{-n^2x^2}}{n} \Rightarrow f_n'(x)=\frac{e^{-n^2x^2}(-2n^2x)}{n}$$

$$\Rightarrow \quad f_n'(x)=-2n\,xe^{-n^2x^2}$$

Also, $\lim_{n\to\infty} f_n(x)= f(x)=\lim_{n\to\infty}\frac{e^{-n^2x^2}}{n}=0\,;\ \forall x\in R$

& $\quad g(x)=\lim_{n\to\infty} f_n'(x)=\lim_{n\to\infty}\frac{-2n\,x}{e^{n^2x^2}}=0;\ \forall x\in R$

Let $p(x)=\left|f_n(x)-f(x)\right|=\dfrac{e^{-n^2x^2}}{n}$

$$\Rightarrow \quad p'(x)=\frac{-2n^2x\,e^{-n^2x^2}}{n}=-2n\,xe^{-n^2x^2}$$

+	0	−

sign scheme of $p'(x)$

$$M_n=\sup_{x\in R} p(x)=p(0)=\frac{1}{n}$$

$$\Rightarrow \quad \lim_{n\to\infty} M_n=\lim_{n\to\infty}\frac{1}{n}=0$$

So, $\{f_n(x)\}$ converges uniformly to $f(x)=0$ in R.

Now, $\{f_n'(x)\}$ converges pointwise to 0 in R.
Further, let

$$q(x)=\left|f_n'(x)-g(x)\right|=2n|x|e^{-n^2x^2}$$

$q(x)$:

$-2nxe^{-n^2x^2}$	0	$2nxe^{-n^2x^2}$

$$h(x)=2\,nxe^{-n^2x^2} \Rightarrow h'(x)=2\,ne^{-n^2x^2}+$$

$$2n\,x(-2\,n^2x)e^{-n^2x^2}$$

$$=2\,n\,e^{-n^2x^2}\left[1-2\,h^2\,x^2\right]=-4n^3e^{-n^2x^2}\left[1-\frac{1}{2n^2}\right]$$

$q'(x)$:

$4n^3e^{-n^2x^2}\left(1-\frac{1}{2n^2}\right)$	0	$\times\ -4n^3e^{-n^2x^2}\left(1-\frac{1}{2n^2}\right)$

So, $\sup_{x\in R} q(x)=\sup_{x\in R}\left|f_n'(x)-g(x)\right|=g\left(\dfrac{1}{\sqrt{2n}}\right)$

$$=g\left(-\frac{1}{\sqrt{2n}}\right)=2n\times\frac{1}{\sqrt{2n}}e^{-n^2\left(\frac{1}{2n^2}\right)}$$

$$=\sqrt{2}\,e^{-\frac{1}{2}}=M_n$$

$\because \quad \lim_{n\to\infty} M_n=\dfrac{\sqrt{2}}{\sqrt{e}}=\sqrt{\dfrac{2}{e}}\neq 0$. So $\{f_n'(x)\}$ do not

converge uniformly in R and point of non uniform convergence is 0.
So, option (2) is correct only.

Q 101. Ans (1,2,3,4)

If $T = [a_{ij}]_{n\times n}$ then $tr(TT^*) = \sum_{i=1}^{n}\sum_{j=1}^{n}|a_{ij}|^2$

So, trace$(TT^*) = 0 \Rightarrow \sum_{i=1}^{n}\sum_{j=1}^{n}|a_{ij}|^2 = 0$

$\Rightarrow$ $a_{ij} = 0$; $\forall i, j \Rightarrow T = 0$

(option 1 is correct)

We know that Rank$(T^*T) =$ Rank (T)

So, Nullity $(T^*T) =$ Nullity (T) (1)

and as $T(V) = 0 \Rightarrow T^*T(V) = 0$, so

$N(T) \subseteq N(T^*T)$ but from (1)

$\ker(T) = \ker(T^*T)$, So

$T^*T(V) = 0 \Rightarrow T(V) = 0$

So, option (2) is correct.

Also, $T = T^* \Rightarrow T$ is Hermitian operator.

$\Rightarrow$ T is normal operator, so it is diagonalisable,

hence $T^{2N}(V) = 0 \Rightarrow T(V) = 0$

$\because$ Rank $(T^{2N}) =$ Rank (T)

$\therefore$ Nullity $(T^{2N}) =$ Nullity (T)

So, option (3) is correct.

Also, $TT^* = T^*T \Rightarrow T$ is normal operator, so T is diagonalisable.

So Rank $(T^N) =$ Rank (T) & hence

Nullity $(T^N) =$ Nullity (T)

$\Rightarrow$ $\ker(T^N) = \ker(T)$

So, $T^N(V) = 0 \Rightarrow T(V) = 0$

So, option (4) is also correct.

Hence all 4 options are correct.

Q 102. Ans (1,4) (Statistics)

Q 103. Ans (2,3,4)

In the given variational problem

$F(y, y') = a(y')^2 + 2b\,y\,y' + c\,y^2$

for strong minima

$F_{y'y'} > 0 \Rightarrow 2a > 0 \Rightarrow a > 0$

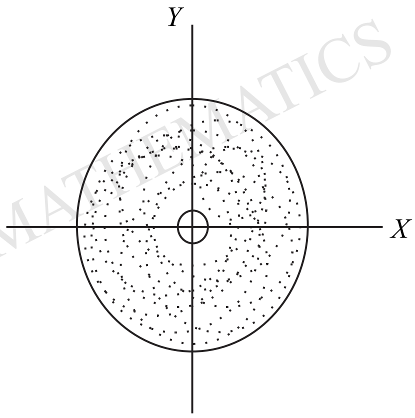
In option (1), $a = -2 < 0$

So, it is false.

In option 2, 3 & 4 $a = 1, 2 \,\&\, 1$ respectively which are greater than 0 so they are true.

Q 104. Ans (1,4) (Statistics)

Q 105: Ans (2,3,4)



$D^\times = \{z \in c : 0 < |z| < 1\}$

$f : D^\times \to D^\times$ such that f is bijective and holomorphic

$\Rightarrow$ $f(z) = e^{i\theta}\dfrac{z-a}{1-\bar{a}z}$ & $f(0) = 0$; $a \in D$

& $\theta \in R$

where $D = \{z \in \mathbb{C} : |z| < 1\}$

Hence, $f(0) = 0 \Rightarrow \dfrac{e^{i\theta}(0-a)}{1-\bar{a}.(0)} = 0 \Rightarrow a = 0$

$\Rightarrow$ $f(z) = e^{i\theta}z$; $\theta \in R$

Also, $\lim_{z\to 0} f(z) = \lim_{z\to 0} e^{i\theta}z = 0$ whose absolute value is 0 which is ≤ 1.

So, options (2), (3) & (4) are correct.

Q 106. Ans (2,3,4)

(1) Let $p=3$, $f(x)=(x+1)(x^2+1)$.

Then $\alpha=-1\equiv 2(\bmod 3)$ is a root of $f(x)$ in $\mathbb{Z}_3$. But $\mathbb{F}_p(\alpha)$ is not splitting field of $f(x)$ in $\overline{\mathbb{F}_p}$,

(2) Given f and g are irreducible polynomial of same degree over $\mathbb{F}_p$.

Let say degree$(f(x))=$ degree $(g(x))=n$ and α be a root of $f(x)$ in $\overline{\mathbb{F}_p}$.

$\Rightarrow$ $\dfrac{\mathbb{F}_p[x]}{\langle f(x)\rangle}\cong\mathbb{F}_p(\alpha)$ is field of order p^n.

Hence $\mathbb{F}_p(\alpha)\cong\mathbb{F}_{p^n}$ contains all root of $g(x)$ in $\overline{\mathbb{F}_p}$

$\Rightarrow$ $\mathbb{F}_p(\alpha)$ is the splitting field of $g(x)$.

(3) (Result)

$\mathbb{Z}_p[x]$ contains each degree irreducible polynomial.

OR

For $n\geq 1$, there exist irreducible polynomial of degree n over $\mathbb{F}_p$.

(4) Every elements of $\mathbb{F}_p$ is a sum of two squares.

So, option (2), (3) & (4) are correct.

Q 107. Ans (1,2,3)

$$F(x,y,y')=\frac{(y')^2}{x^\alpha}$$

By Euler equation

$$\frac{\partial F}{\partial y}-\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)=0$$

$$\Rightarrow\quad 0-\frac{d}{dx}\left(\frac{2y'}{x^\alpha}\right)=0\Rightarrow\frac{d}{dx}\left(\frac{2y'}{x^\alpha}\right)=0$$

$$\Rightarrow\quad \frac{2y'}{x^\alpha}=c\Rightarrow y'=\frac{cx^\alpha}{2}\Rightarrow\int dy=\int\frac{cx^\alpha}{2}dx+c_2$$

$$\Rightarrow\quad y=c_1\frac{x^{\alpha+1}}{\alpha+1}+c_2 \qquad (1)$$

$$y(1)=1\Rightarrow\frac{c_1}{\alpha+1}+c_2=1 \qquad (2)$$

$$y(2)=2\Rightarrow\frac{c_1.2^{\alpha+1}}{\alpha+1}+c_2=2 \qquad (3)$$

$$(3)\text{-}(2)\Rightarrow\frac{c_1}{\alpha+1}\left(2^{\alpha+1}-1\right)=1\Rightarrow c_1=\frac{\alpha+1}{2^{\alpha+1}-1} \qquad (4)$$

$$\Rightarrow\quad c_2=1-\frac{c_1}{\alpha+1}=1-\frac{1}{2^{\alpha+1}-1}=\frac{2^{\alpha+1}-2}{2^{\alpha+1}-1}$$

$\Rightarrow$ $y=\dfrac{x^{\alpha+1}+2^{\alpha+1}-2}{2^{\alpha+1}-1}$ is extremal of $J_\alpha[y]$

So, extremal for J_3 is $(\alpha=3)$

$y(x)=\dfrac{x^4+14}{15}$. So option (1) is correct

extremal for J_1 is $(\alpha=1)$

$\dfrac{(x^2+2)}{3}$. So, option (2) is correct extremal for J_0 is $(\alpha=0)$

x. So, option (3) is correct.

Option (4) is false as option (2) is true.

Q 108. Ans (1,3) (Statistics)

Q 109. Ans (1,3) (Statistics)

Q 110. Ans (1,2,4)

Given $y''-3y'+2y=4\sin x$

$y(0)=1, y'(0)=-2$

$$\Rightarrow\quad \int_0^x\int_0^x y''\,dx\,dx-3\int_0^x\int_0^x y'\,dx\,dx+$$

$$2\int_0^x\int_0^x y\,dx\,dx=4\int_0^x\int_0^x\sin x\,dx\,dx$$

$$\Rightarrow\quad \int_0^x\left(y'(x)+2\right)dx-3\int_0^x\left(y(x)-1\right)dx$$

$$+2\int_0^x\int_0^x y\,dx\,dx=4\int_0^x(1-\cos x)\,dx$$

$$\Rightarrow\quad y(x)-1+2x+3x-3\int_0^x y(x)dx$$

$$+2\int_0^x\int_0^x y\,dx\,dx=4(x-\sin x)$$

$\Rightarrow \quad y(x)=1-5x+3\int_0^x y(x)\,dx-2\int_0^x\int_0^x y\,dx\,dx$

$+4(x-\sin x)$

$\Rightarrow \quad y(x)=1-x-4\sin x+\int_0^x\left(3-2\int_0^x y\,dx\right)dx$

$\Rightarrow \quad f(x)=1-x-4\sin x$

$\Rightarrow \quad f'(x)=-1-4\cos x$

$\Rightarrow \quad f'(\pi)=-1-4\cos\pi=-1+4=3$

Option (1) is correct.

$f(\pi)=1-\pi-4\sin\pi=1-\pi$

$\Rightarrow \quad f(\pi)+f'(\pi)=1-\pi+3=4-\pi$

So, option (2) is correct & Option (3) is false.

$f(0)=1-0-4\sin 0=1$

$f''(0)=-1-4\cos 0=-1-4=-5$

So, $f(0)+f'(0)=1+(-5)=-4$

So, option (4) is correct.

Q 111. Ans (2,4)

$B(v,w)$ is non degenerate symmetric bilinear form on R^2 and we have $q(v)=B(v,v)$

$\exists v,w\in R^2$ such that $B(v,v)=0$ & $B(v,w)\neq 0$

$\Rightarrow \quad v\neq 0$ & $w\neq 0$ & w is not multiple of v, so $\{v,w\}$ is a basis of R^2

Matrix of bilinear form is

$$A=\begin{bmatrix}B(v,v) & B(v,w)\\ B(v,w) & B(w,w)\end{bmatrix}=\begin{bmatrix}0 & k\\ k & b\end{bmatrix}$$

where $B(w,w)=b$ & $B(v,w)=k\neq 0$

$|A|=-k^2 \Rightarrow A$ has one eigenvalue positive and one eigenvalue negative.

Hence quadratic form q is indefinite

So, q is equivalent to quadratic form $Q(x,y)=x^2-y^2\ \forall (x,y)\in R^2$.

Option (4) is correct.

$q(\alpha v+w)=B(\alpha v+w,\alpha v+w)$

$=\alpha^2 B(v,v)+\alpha B(v,w)+\alpha B(w,v)+B(w,w)$

$=2\alpha B(v,w)+B(w,w)=0$

$\Rightarrow \quad \alpha=\dfrac{-B(w,w)}{2B(v,w)}\neq 0$

So, α is unique hence, option (2) is correct but option (3) is incorrect.

Q 112. Ans (1)

$xu_x+u_y=1$ has auxiliary equation as

$\dfrac{dx}{x}=\dfrac{dy}{1}=\dfrac{du}{1}$

now, $\dfrac{dx}{x}=\dfrac{dy}{1}\Rightarrow \ln x=y+c\Rightarrow x=c_1e^y$

$\Rightarrow \quad xe^{-y}=c_1$ (1)

also, $\dfrac{dy}{1}=\dfrac{du}{1}\Rightarrow du=dy\ \Rightarrow u=y+c_2$

$\Rightarrow \quad u-y=c_2$ (2)

(1) & (2) $\Rightarrow u-y=\phi\left(xe^{-y}\right)$

$\Rightarrow \quad u=y+\phi\left(xe^{-y}\right)$

Now $u(0,y)=e^y$

$\Rightarrow \quad e^y=y+\phi(0)\Rightarrow \phi(0)=e^y-y$

which is impossible so there is no neighbourhood of the origin on which (CP) has a solution.

Q 113. Ans (1,3)

$\because \quad \{a_n\}_0^\infty$ is unbounded strictly increasing sequence, so it diverges to ∞.

Also $a_{n+1}\leq ca_n \Rightarrow \dfrac{a_{n+1}}{a_n}\leq c$

also $\dfrac{a_{n+1}}{a_n}>1\Rightarrow 1<\dfrac{a_{n+1}}{a_n}\leq c$; $\forall n\geq 1$

Also $A_k = \{x \in R \mid a_k \le f(x) < a_{k+1}\}$

(1) If f is Lebesgue integrable on R then $\int_R f(x)\,dx < \infty$ i.e. it is finite.

So, $\sum_{k\ge 0} a_k \mu(A_k)$ is finite as it is lower lebesgue sum which must be finite.
So, option (1) is correct.

(2) If we take

$$f(x) = \sin\frac{1}{x} \;;\; x \in (0,1)$$
$$= 0 \;;\; \text{otherwise}$$

then $f(x)$ is not Lebesgue integrable in R but $\sum_{k\ge 0} a_k \mu(A_k)$ is finite & $f(x)$ is bounded.
So, option (2) & (4) are incorrect.
Also if $f(x) \ge a_1 \;;\; \forall x \in R$ and $\sum_{k\ge 0} a_k \mu(A_k)$ is finite then $\int_R f(x)\,dx < \infty$ i.e. it is Lebesgue integrable.So, option (3) is correct.

Q 114. Ans (1,2)

$$S = \{a + b\sqrt{2} \mid a, b \in Q\} = Q(\sqrt{2})$$

$\Rightarrow$ derived set of $S = R$ so $\tilde{S} = R$

$\Rightarrow$ S is dense in R

Also $(S \setminus Q)' = R \Rightarrow (\tilde{S} \setminus Q) = R$

So, $S \setminus Q$ is also dense in R.
So, option (1) & (2) are correct.
As S is not an interval, so S is not connected and $S' = \phi$ implies that it is not discrete.
So, Option (3) & (4) are false.

Q 115. Ans (2)

$$\lim_{n\to\infty} \frac{1}{n}\left[(n+1)(n+2)...(n+n)\right]^{1/n} = L \text{ (let)}$$

$$= \lim_{n\to\infty}\left[\frac{(n+1)(n+2)...(n+n)}{n^n}\right]^{1/n} \qquad (1)$$

$$= \lim_{n\to\infty}(u_n)^{1/n} \;;\; u_n = \frac{(n+1)(n+2)...(n+n)}{n^n}$$

$$\Rightarrow \quad u_{n+1} = \frac{(n+2)(n+3)...(2n)(2n+1)(2n+2)}{(n+1)^{n+1}}$$

So, by Cauchy's Second theorem on limit.

$$L = \lim_{n\to\infty}\frac{u_{n+1}}{u_n} = \lim_{n\to\infty}\frac{(2n+1)(2n+2)}{(n+1)^2\left(1+\frac{1}{n}\right)^n} = \frac{4}{e}$$

Q 116. Ans (2,3)

$$y_+ = \max\{y, 0\} = 0 \;;\; -1 \le y \le 0$$
$$= y \;;\; 0 \le y \le 1$$

$$\Rightarrow \quad f(x,y) = 1 \;;\; -1 \le y \le 0$$
$$= 1 + \sqrt{y} \;;\; 0 \le y \le 1$$

Now

$$|f(x_1 y_1) - f(x_1 y_2)| = \left|(1+\sqrt{y_1}) - (1+\sqrt{y_2})\right|$$

$$= \left|\sqrt{y_1} - \sqrt{y_2}\right| = \frac{|y_1 - y_2|}{\left|\sqrt{y_1} + \sqrt{y_2}\right|} \to \infty |y_1 - y_2|$$

if $y_1, y_2 \to 0^+$. So it is not a Lipschitz continuous on D.
So, option (1) is false and option (2) is true.
As $f(x,y)$ is continuous and bounded in given domain D, so IVP has at least one solution. So option (3) is true and option (4) is false.

Q 117. Ans (1,2,3,4) (Mechanics)

Q 118. Ans (1,2,4)

$$W(\phi_1, \phi_2)\Big|_0 = \begin{vmatrix} \phi_1(0) & \phi_2(0) \\ \phi_1'(0) & \phi_2'(0) \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \ne 0$$

$\Rightarrow$ ϕ_1 & ϕ_2 are L.I.

$\Rightarrow$ $y = C_1\phi_1(x) + C_2\phi_2(x)$ is general solution.

By Abel's formula

$$W(x) = W(x_0)e^{-\int_{x_0}^{x} P\,dx}$$

$$= W(0)e^{-\int_0^x \cos x\,dx} = e^{-\sin x}$$

$$\ne 0 \;;\; \forall x$$

As coefficient of y'', y' & y i.e.

$1, \cos x \,\&\, \sin x$ are period function and each of them has a period $2\pi \,\&\, 4\pi$

So, $\phi_1(x+2\pi)$ is also a solution of ODE

(Option 1 is correct)

$\phi_2(x+4\pi)$ is also a solution of ODE.

(Option 2 is correct)

Also $\exists a, b \in R$ such that

$\phi_1(x+2\pi) = a\phi_1(x) + b\phi_2(x)$ i.e. it is spanned by L.I. solutions $\phi_1(x) \,\&\, \phi_2(x)$.

So, option (4) is correct

But option (3) is false

$\because$ $\phi_2(x+4\pi)$ is also spanned by $\phi_1(x) \& \phi_2(x)$

Q 119. Ans (4)

$\gamma : [0,1] \to C$ is given by $t \mapsto e^{2\pi i t}$

$\Rightarrow$ $\gamma(t) = e^{2\pi i t}$; $t \in [0,1]$

$\Rightarrow$ $|\gamma(t)| = |e^{2\pi i t}| = 1 \,\&\, 2\pi t \in [0, 2\pi]$

So, it is complete one round of the circle $C : |z| = 1$

So, $I = \int_\gamma e^z e^{\frac{1}{z}} dz = 2\pi i \operatorname*{Res}_{z=0} e^z . e^{\frac{1}{z}}$

$$= 2\pi i \operatorname*{Res}_{z=0} \left(1 + \frac{z}{1!} + \frac{z^2}{2!} + \ldots\right)\left(1 + \frac{1}{z.1!} + \frac{1}{z^2.2!} + \ldots\right)$$

$$= 2\pi i \left[\text{coefficient of } \frac{1}{z} \text{in} \left(1 + \frac{z}{1!} + \frac{z^2}{2!} + \ldots\right) \left(1 + \frac{1}{z.1!} + \frac{1}{z^2.2!} + \ldots\right)\right]$$

$$= 2\pi i \left(\frac{1}{0!.1!} + \frac{1}{1!.2!} + \frac{1}{2!.3!} + \ldots\right)$$

$$= 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!} \;.$$

So, option (4) is correct.

Q 120. Ans (1,3) (Statistics)

****